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# Existence of solutions of nonlinear differential equations with generalized dichotomous linear part in a Banach space

ABSTRACT. A generalization of the well known dichotomies for a class of homogeneous differential equations in an arbitrary Banach space is introduced. The aim of this paper is the consideration of the nonlinear differential equation with generalized dichotomous linear part. By the help of the fixpoint principle of Banach and Schauder-Tychonoff are found sufficient conditions for the existence of solutions of the nonlinear equation.

KEY WORDS. Ordinary Differential Equations, Generalized Dichotomy

## 1 Introduction

The notion of exponential and ordinary dichotomy is fundamental in the qualitative theory of ordinary differential equations. It is considered in detail for example in the monographs [2], [3],[6–8].

In the given paper we use a $(M, N, R)$ dichotomy, introduced in [5], which is a generalization of all dichotomies known by the authors.

It is considered a nonlinear differential equation with generalized dichotomous linear part. A nonlinear operator, acting in the phase space is introduced. Sufficient conditions for the existence of fixed point of this operator are found. These fixed points are solutions of the differential equation.

## 2 Problem statement

Let $X$ is an arbitrary Banach space with norm $|.|$ and identity $I$ and let $J = [c, \infty)$ where $c \in \mathbb{R}$. Let $L(X)$ is the space of all linear bounded operators acting in $X$ with the norm $||.||$.

We consider the nonlinear differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = A(t)x + F(t, x), \tag{1}$$

where $A(.) : J \to L(X),\ F(.,.) : J \times X \to X$. Let $F$ is continuous.

By $V(t)$ we will denote the Cauchy operator of

$$\frac{\mathrm{d}x}{\mathrm{d}t} = A(t)x \tag{2}$$

where $A(t) \in L(X), t \in J$.

We consider also the nonhomogeneous equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = A(t)x + f(t) \tag{3}$$

where $f(.) : J \to X$ is continuous and bounded.

In this paper we will use the $(M, N, R)$-dichotomy, introduced in [5] with both following theorems.

Let $R(t) : X \to X \quad (t \in J)$ is an arbitrary bounded operator.

**Lemma 1** [5] *The function*

$$x(t) = \int_c^t V(t)R(s)V^{-1}(s)f(s)\mathrm{d}s - \int_t^\infty V(t)(I - R(s))V^{-1}(s)f(s)\mathrm{d}s \tag{4}$$

*is a solution of the equation (3) if the integrals in (4) exist.*

Following conditions are introduced

H1. $\mid V(t)R(s)V^{-1}(s)z \mid \leq M(t, s, z), t \geq s, z \in X$

H2. $\mid V(t)(I - R(s))V^{-1}(s)z \mid \leq N(t, s, z), t < s, z \in X$

For many important cases the right hand part of (H1) and (H2) has the form

$$\begin{cases} M(t, s, z) = \varphi_1(t)\varphi_2(s) \mid z \mid, \ (t \geq s), \ z \in X \\ N(t, s, z) = \psi_1(t)\psi_2(s) \mid z \mid, \ (t < s), \ z \in X \end{cases} \tag{5}$$

where $\varphi_1(t), \varphi_2(t), \psi_1(t), \psi_2(t)$ are positive scalar functions. We set

$$\alpha(t) = \max\{\varphi_1(t), \psi_1(t)\},$$

$$\mu(t) = \min\{\varphi_1(t), \psi_1(t)\},$$

$$\beta(t) = \max\{\varphi_2(t), \psi_2(t)\} \quad (t \in J).$$

**Definition 1** *We call the equation (2) be a $(M, N, R)$ - dichotomous if the conditions (H1), (H2) are fulfilled.*

Let $a(t)$ is an arbitrary positive scalar function. We consider the following Banach spaces :

$$K_a = \{g : J \to X : \sup_{t \in J} a(t) \int_c^t M(t, s, g(s))\mathrm{d}s < \infty\}$$

with the norm

$$\mid g \mid_{K_a} = \sup_{t \in J} \ a(t) \int_c^t M(t, s, g(s))\mathrm{d}s,$$

$$L_a = \{g : J \to X : \sup_{t \in J} a(t) \int_t^\infty N(t, s, g(s))\mathrm{d}s < \infty\}$$

with the norm

$$\mid g \mid_{L_a} = \sup_{t \in J} \ a(t) \int_t^\infty N(t, s, g(s))\mathrm{d}s,$$

$$C_a = \{g : J \to X : \sup_{t \in J} a(t) \mid g(t) \mid < \infty\}$$

with the norm

$$\mid g \mid_{C_a} = \sup_{t \in J} \ a(t) \mid g(t) \mid$$

and

$$T_a = \{g : J \to X : \int_c^\infty a(s) \mid g(s) \mid \mathrm{d}s < \infty\}$$

with the norm

$$\mid g \mid_{T_a} = \int_c^\infty a(s) \mid g(s) \mid \mathrm{d}s.$$

The case, when $X = \mathbb{R}_+$ will be denoted with $\bar{T}_a$ :

$$\bar{T}_a = \{g : J \to \mathbb{R}_+ : \int_c^\infty a(s)g(s)\mathrm{d}s < \infty\}$$

with the norm

$$\mid g \mid_{\bar{T}_a} = \int_c^\infty a(s)g(s)\mathrm{d}s.$$

**Theorem 1** [5] *Let the equation* (2) *is* $(M, N, R)$ *- dichotomous. Then for every function* $f \in K_a \bigcap L_a$ *the equation* (3) *has a solution in the space* $C_a$.

**Corollary 1** [5] *Let the equation* (1) *is* $(M, N, R)$ *- dichotomous of the form* (5).

*Then for every function* $f \in K_{\mu^{-1}} \bigcap L_{\mu^{-1}}$ *the equation* (2) *has a solution in the space* $C_{\alpha^{-1}}$ *and the following estimates hold*

$$\sup_{t \in J} \ \alpha^{-1}(t) \mid x(t) \mid \leq \int_c^t \beta(s) \mid f(s) \mid \mathrm{d}s + \int_t^\infty \beta(s) \mid f(s) \mid \mathrm{d}s < \infty$$

**Theorem 2** [5] *Let the equation* (2) *is* $(M, N, R)$ *- dichotomous.*

*Then following estimates hold*

$$| x_1(t) | \leq M(t, s, x_1(s)), \ t \geq s \geq c \tag{6}$$

*for all solutions* $x_1(t)$ *of (1)* ,$(t \geq c)$, *which started in the set*

$$\bigcap_{s \in J} Fix\ R(s)$$

*and*

$$| x_2(t) | \leq N(t, s, x_2(s)), \ c \leq t < s \tag{7}$$

*for all solutions* $x_2(t)$ *of (1)* ,$(t \geq c)$, *which started in the set*

$$\bigcap_{s \in J} Fix(I - R(s))$$

(By $FixS$ we denote the set of all fixed points of the map $S$, $S : X \to X$.)

**Remark 1** Let $R(t) = P$,where $P : X \to X$ is a projector.

For

$$M(t, s, z) = K_1 e^{-\int_s^t \delta_1(\tau) \mathrm{d}\tau} |z| \ (t \geq s, z \in X)$$
$$N(t, s, z) = K_2 e^{-\int_t^s \delta_1(\tau) \mathrm{d}\tau} |z| \ (s > t, z \in X)$$

where $K_1, K_2$ are positive constants and $\delta_1, \delta_2$ are continuous real-valued functions on $J$, we obtain the exponential dichotomy of [7]:

$$\| V(t) P V^{-1}(s) \| \leq K_1 e^{-\int_s^t \delta_1(\tau) \mathrm{d}\tau} \ (t \geq s)$$

$$\| V(t)(I - P) V^{-1}(s) \| \leq K_2 e^{-\int_t^s \delta_2(\tau) \mathrm{d}\tau} \ (s > t).$$

For $\delta_i(t) = 0$ $(c \leq t < \infty, \ i = 1, 2)$ we obtain the exponential dichotomy of [2], [3], [6], for which case we have $K_a \bigcap L_a = C_a$ by $a(t) \equiv 1$.

For

$$M(t, s, z) = K h(t) h^{-1}(s) |z| \ (t \geq s \geq c, , z \in X)$$
$$N(t, s, z) = K k(t) k^{-1}(s) |z| \ (c \leq t \leq s, z \in X)$$

where $K$ is a positive constant and $h, k : [0, \infty) \to (0, \infty)$ are two continuous functions, we obtain the dichotomy of [8–10]:

$$\| V(t) P V^{-1}(s) \| \leq K h(t) h^{-1}(s) \ , (t \geq s \geq c)$$

$$\| V(t)(I - P) V^{-1}(s) \| \leq K k(t) k^{-1}(s) \ , (c \leq t \leq s)$$

It may be also noted, that the dichotomies [1], [7–10] are a generalization of the dichotomy in [3].

## 3 Main results

By the help of the fixpoint principle of Banach we will find sufficient conditions for the existence of solutions of the nonlinear equation (1).

Let $r > 0$. We introduce following conditions

H3. There exists a positive function $m \in \bar{T}_\beta$, such that

$$|F(t,x)| \le m(t) \quad (|x| \le r,\ t \in J).$$

H4. There exists a positive function $k \in \bar{T}_\beta$, such that

$$|F(t,x_2) - F(t,x_1)| \le \alpha^{-1}(t)k(t)|x_2 - x_1| \quad (|x_1|, |x_2| \le r,\ t \in J).$$

We set $a_1 = |m|_{\bar{T}_\beta},\ a_2 = |k|_{\bar{T}_\beta}$.

**Definition 2** *We say that the equation (1) belongs to the class $D(a_1, a_2, r)$ if there exists $r > 0$, such that the conditions (H3) and (H4) are fulfilled.*

**Theorem 3** *Let the linear part of (1) is $(M, N, R)$ dichotomous with $R(s)$ $(s \in J)$ be linear and the conditions (H1) and (H2) have the form (5).*

*Then there exist numbers $\bar{a}_1, \bar{a}_2 > 0$ and $\rho < r$ with following property:*

*If the initial value $\xi$ fulfilled $|\xi| \le \rho$ and if the equation (1) belongs to the class $D(a_1, a_2, r)$ for $a_1 \in (0, \bar{a}_1),\ a_2 \in (0, \bar{a}_2)$ then there exists an unique solution $x(t)$ in the ball $|\, x\,|_{C_{\alpha^{-1}}} \le r$, i.e.*

$$\sup_{t \in J} \alpha^{-1}(t)|x(t)| \le r$$

**Proof:** First we shall prove, that the operator $Q$, defined by the formula

$$(Qx)(t) = V(t)R(c)\xi + \int_c^t V(t)R(s)V^{-1}(s)F(s,x(s))\mathrm{d}s -$$

$$- \int_t^\infty V(t)(I - R(s))V^{-1}(s)F(s,x(s))\mathrm{d}s$$

maps the ball $|\, x\,|_{C_{\alpha^{-1}}} \le r$ into itself. Indeed we have

$$|(Qx)(t)| \le \varphi_1(t)\varphi_2(c)|\xi| + \int_c^t \varphi_1(t)\varphi_2(s)m(s)\mathrm{d}s + \int_t^\infty \psi_1(t)\psi_2(s)m(s)\mathrm{d}s$$

$$|(Qx)(t)| \le \alpha(t)\varphi_2(c)|\xi| + \alpha(t)\int_c^\infty \beta(s)m(s)\mathrm{d}s$$

Hence

$$\alpha^{-1}(t)|(Qx)(t)| \leq \varphi_2(c)\rho + a_1$$

For sufficiently small $\rho$ and $a_1$, $Q$ will map the ball $|\, x\, |_{C_{\alpha^{-1}}} \leq r$ into itself.

Now we shall prove, that the operator $Q$ is a contraction in the ball $|\, x\, |_{C_{\alpha^{-1}}} \leq r$

Indeed, we have

$$\begin{aligned}
|(Qx_1)(t)-(Qx_2)(t)| \leq & \int_c^t |V(t)R(s)V^{-1}(s)(F(s,x_1(s)) - F(s,x_2(s)))|\mathrm{d}s+ \\
& + \int_t^\infty |V(t)(I - R(s))V^{-1}(s)(F(s,x_1(s)) - F(s,x_2(s)))|\mathrm{d}s \leq \\
& \leq \int_c^t \varphi_1(t)\varphi_2(s)|F(s,x_1(s)) - F(s,x_2(s))|\mathrm{d}s+ \\
& + \int_t^\infty \psi_1(t)\psi_2(s)|F(s,x_1(s)) - F(s,x_2(s))|\mathrm{d}s \leq \\
& \leq \alpha(t) \int_c^\infty \beta(s)\alpha^{-1}(s)k(s)|x_1(s) - x_2(s)|\mathrm{d}s
\end{aligned}$$

We obtain

$$\alpha^{-1}(t)|(Qx_1)(t) - (Qx_2)(t)| \leq \sup_{t\in J} \alpha^{-1}(t)|x_1(t) - x_2(t)| \int_c^\infty \beta(s)k(s)\mathrm{d}s$$

$$|Qx_1 - Qx_2|_{C_{\alpha^{-1}}} \leq |x_1 - x_2|_{C_{\alpha^{-1}}} |k|_{T_{\bar\beta}} = |x_1 - x_2|_{C_{\alpha^{-1}}} a_2$$

Hence for sufficiently small $a_2$, the operator $Q$ is a contraction in the ball $|\, x\, |_{C_{\alpha^{-1}}} \leq r$.

The assertion of the theorem follows from the theorem of Banach - Cacciopolli [4]. □

Other sufficient conditions for existence of solution of the equation (1) we will find, using the fixed point principle of Schauder-Tychonoff. In connection with its applying, we will use a generalization of the Arzella-Ascoli's theorem for locally convex spaces.

Let $S(J,X)$ is the linear set of all functions, acting from $J$ in $X$, which are continuous. The set $S(J,X)$ is a locally convex space w.r.t. the metric

$$\rho(u,v) = \sup_{c<T<\infty} (1+T)^{-1} \frac{\max\limits_{c\leq t\leq T} \|u(t) - v(t)\|}{1 + \max\limits_{c\leq t\leq T} \|u(t) - v(t)\|}.$$

The convergence with respect to this metric coincides with the uniform convergence on each bounded interval. For this space an analog of Arzella-Ascoli's theorem is valid.

**Lemma 2** *The set $H \subset S(J,X)$is relatively compact if the intersections $H(t) = \{h(t) : h \in H\}$ are relatively compact subsets of $X$ for every $t \in J$ and $H$ is equicontinuous on each finite closed interval.*

**Proof:** We apply Arzella-Ascoli's theorem to each finite and closed interval. □

Let $C$ is an unempty subset of $X$ and let

$$\tilde{C} = \{u \in S(J, X) : u(t) \in C, t \in J\}$$

**Lemma 3** *Let $C$ is an unempty, convex and closed subset of $X$ and the operator $F$ maps $\tilde{C}$ into itself and is continuous. Let $F(\tilde{C})$ is relatively compact subset of $\tilde{C}$.*

*Then $F$ has a fixed point in $\tilde{C}$.*

**Proof:** It follows from the fixed point principle of Schauder-Tychonoff [4]. □

Let

$$C(r) = \{x \in S(J, X) : |x|_{C_{\alpha^{-1}}} \leq r\}$$

Obviously $C(r)$ is unempty, convex and closed.

**Theorem 4** *Let the following conditions are fulfilled:*

1. *Let the linear part of (1) is $(M, N, R)$ dichotomous and the conditions (H1) and (H2) have the form (5).*

2. *There exists a number $r > 0$ such that*

$$\sup_{|u| \leq r} |F(t, u)| = m(t), \text{ where } \; m \in \bar{T}_\beta.$$

3. *The function $F(t, u)$ is continuous $(t \in J, |u| \leq r)$.*

4. *The set $K(r) = \{m^{-1}(t)F(t, x) : t \in J, |u| \leq r\}$ is relatively compact.*

5. *$R(t)u$ is continuous for every $u \in X$ by any fixed $t \in J$.*

*Then for sufficient small $|m|_{\bar{T}_\beta}$ the nonlinear equation (1) has a solution $x \in C(r)$ for which*

$$x(c) = R(c)\xi - \int_c^\infty (I - R(s))V^{-1}(s)F(s, x(s))\mathrm{d}s \quad, (|\xi| \leq r)$$

**Proof:** We consider the operator $Q$ defined by the formula

$$(Qx)(t) = V(t)R(c)\xi + \int_c^t V(t)R(s)V^{-1}(s)F(s, x(s))\mathrm{d}s -$$

$$- \int_t^\infty V(t)(I - R(s))V^{-1}(s)F(s, x(s))\mathrm{d}s,$$

where ($|\xi| \leq r$). First we shall prove, that $Q$ maps $C(r)$ into itself. Let $x \in C(r)$. Then

$$|(Qx)(t)| \leq \varphi_1(t)\varphi_2(c)|\xi| + \int_c^t \varphi_1(t)\varphi_2(s)m(s)\mathrm{d}s + \int_t^\infty \psi_1(t)\psi_2(s)m(s)\mathrm{d}s$$

$$|(Qx)(t)| \leq \varphi_1(t)\varphi_2(c)|\xi| + \varphi_1(t)\int_c^t \varphi_2(s)m(s)\mathrm{d}s + \psi_1(t)\int_t^\infty \psi_2(s)m(s)\mathrm{d}s$$

$$|(Qx)(t)| \leq \alpha(t)\varphi_2(c)|\xi| + \alpha(t)\int_c^\infty \beta(s)m(s)\mathrm{d}s$$

Hence

$$\alpha^{-1}(t)|(Qx)(t)| \leq \varphi_2(c)|\xi| + |m|_{\bar{T}_\beta}$$

For sufficiently small $|\xi|$ and $|m|_{\bar{T}_\beta}$ we obtain $\alpha^{-1}(t)|(Qx)(t)| \leq r \ \ (t \in J)$, i.e. $Q$ maps $C(r)$ into itself.

Now we shall prove that the set $QC(r)$ is relatively compact in $S(J, X)$. For this aim we shall show, that the functions of $QC(r)$ are equicontinuous on each finite closed interval $[a, b]$.

Let $a$ and $b$ are fixed and $t', t'' \in [a, b], \ t' < t''$. Then for $x \in C(r)$ we have

$$|(Qx)(t') - (Qx)(t'')| \leq I_1 + I_2 + I_3$$

where

$$I_1 = |V(t')R(c)\xi - V(t'')R(c)\xi|$$

$$I_2 = |\int_c^{t'} V(t')R(s)V^{-1}(s)F(s, x(s))\mathrm{d}s - \int_c^{t'} V(t'')R(s)V^{-1}(s)F(s, x(s))\mathrm{d}s -$$
$$- \int_{t'}^{t''} V(t'')R(s)V^{-1}(s)F(s, x(s))\mathrm{d}s \ |$$

$$I_3 = |\int_{t'}^\infty V(t')(I - R(s))V^{-1}(s)F(s, x(s))\mathrm{d}s -$$
$$- \int_{t''}^\infty V(t'')(I - R(s))V^{-1}(s)F(s, x(s))\mathrm{d}s \ |$$

For $t'' \to t'$ we have $I_1, I_2 \to 0$, because V(t) is continuous in respect to $t$. For $I_3$ we obtain the estimate

$$\begin{aligned} I_3 \leq \int_{t'}^{\infty} &|V(t')(I - R(s))V^{-1}(s)F(s, x(s)) - \\ &- V(t'')(I - R(s))V^{-1}(s)F(s, x(s))|\mathrm{d}s + \\ &+ \int_{t'}^{t''} |V(t'')(I - R(s))V^{-1}(s)F(s, x(s))|\mathrm{d}s \end{aligned} \tag{8}$$

For $t'' \to t'$ the second integral in (8) converges to zero. We will use the Lebesgue's theorem to prove, that the first integral in (8) by $t'' \to t'$ converges to zero too. Because V(t) is continuous in respect to $t$ we have

$$|V(t')(I - R(s))V^{-1}(s)F(s, x(s)) - V(t'')(I - R(s))V^{-1}(s)F(s, x(s))| \underset{t''\to t'}{\longrightarrow} 0$$

From the estimates

$$\begin{aligned}
&\int_c^\infty |V(t')(I - R(s))V^{-1}(s)F(s, x(s))|\mathrm{d}s+ \\
&+ \int_c^\infty |V(t'')(I - R(s))V^{-1}(s)F(s, x(s))|\mathrm{d}s \le \\
&\le \int_c^\infty \varphi_1(t')\varphi_2(s)|F(s, x(s))|\mathrm{d}s + \int_c^\infty \psi_1(t'')\psi_2(s)|F(s, x(s))|\mathrm{d}s \le \\
&\le \int_c^\infty \alpha(t')\beta(s)m(s)\mathrm{d}s + \int_c^\infty \alpha(t'')\beta(s)m(s)\mathrm{d}s \le \\
&\le (\alpha(t') + \alpha(t''))|m|_{\bar{T}_\beta}
\end{aligned}$$

and from the Lebesgue's theorem follows, that the first integral in (8) converges to zero.

Let $t \in [a, b]$ be fixed. We shall show, that the set $(Qx)(t)$ $(x \in C(r))$ is relatively compact in $S(J, X)$.

Let $\epsilon > 0$ be an arbitrary number. If the numbers $T$ and $N$ are large enough, we obtain the inequality

$$|\int_c^\infty W(t, s)F(s, x(s))\mathrm{d}s - \int_c^T W(t, s)F_N(s, x(s))\mathrm{d}s| < \epsilon$$

where

$$W(t, s) = \begin{cases} V(t)R(s)V^{-1}(s) & t \ge s \\ V(t)(I - R(s))V^{-1}(s) & t < s \end{cases}$$

and

$$F_N(t, u) = \begin{cases} m(t) & t \ge s \\ 0 & t < s \end{cases}$$

From condition 4 of the Theorem follows, that for $F(s, x(s)) \in NK$ we have the inclusion

$$\int_c^T W(t, s)F(s, x(s))\mathrm{d}s \in TN \bigcup_{c \le s \le T} W(t, s)K \tag{9}$$

The set in the right hand of (9) is compact. Hence the set

$$\{\int_c^T W(t, s)F(s, x(s))\mathrm{d}s \;:\; x \in C(r)\}$$

is compact too. From the theorem of Hausdorff follows the compactness of the set

$$\{\int_c^\infty W(t,s)F(s,x(s))\mathrm{d}s \ : \ x \in C(r)\}$$

Hence the set $QC(r)$ is relatively compact in $S(J,X)$.

Now we shall prove that the operator $Q$ is continuous.

Let $\{z_n(t)\} \subset C(r)$ is an arbitrary sequence which converges to $z(t)$ in $S(J,X)$ and let $t \in J$ is fixed. Then

$$\begin{aligned} |(Qz)(t) - (Qz_n)(t)| \le \int_c^t & |V(t)R(s)V^{-1}(s)F(s,z(s)) - \\ & - V(t)R(s)V^{-1}(s)F(s,z_n(s))|\mathrm{d}s + \\ & + \int_t^\infty |V(t)(I-R(s))V^{-1}(s)F(s,z(s)) - \\ & - V(t)(I-R(s))V^{-1}(s)F(s,z_n(s))|\mathrm{d}s \end{aligned} \tag{10}$$

Because $F$ and $V(t)R(s)V^{-1}(s)$ are continuous, the first integral in (10) converges to zero, by $n \to \infty$.

Let

$$J_1(s) = |V(t)(I-R(s))V^{-1}(s)F(s,z(s)) - V(t)(I-R(s))V^{-1}(s)F(s,z_n(s))|$$

Because $V(t)(I-R(s))V^{-1}(s)$ is continuous, so we have

$$J_1(s) \underset{n\to\infty}{\longrightarrow} 0 \ \text{ for any } s \ge t.$$

From the estimate

$$\int_c^\infty J_1(s)\mathrm{d}s \le \int_c^\infty \psi_1(t)\psi_2(s)m(s)\mathrm{d}s \le \alpha(t)|m|_{\bar{I}_\beta}$$

and the Lebesgue's theorem follows, that the second integral in (10) converges to zero for $n \to \infty$. Because $QC(r)$ is compact it follows, that

$$Qz_n \underset{n\to\infty}{\longrightarrow} Qz \ \text{ in } S(J,X).$$

From the Schauder-Tychonoff theorem [4] it follows the existence of a fixpoint $x$ of the operator $Q$ in the set $C(r)$. □

**Remark 2** By $dimX < \infty$ the condition 4 of Theorem 4 is not necessary.

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