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On the dual König property of the order-interval hypergraph of a new class of poset

ABSTRACT. Let P be a finite poset. We consider the hypergraph $\mathcal{H}(P)$ whose vertices are the elements of P and whose edges are the maximal intervals of P. It is known that $\mathcal{H}(P)$ has the König and dual König properties for the class of series-parallel posets. Here we introduce a new class which contains series-parallel posets and for which the dual König property is satisfied. For the class of N-free posets, again a generalization of series-parallel posets, we give a counterexample to see that the König property is not satisfied.

1 Introduction

Let P be a finite poset. A subset I of P of the form $I = \{v \in P : p \leq v \leq q\}$ (denoted $[p, q]$) is called an interval. It is maximal if p (resp. q) is a minimal (resp. maximal) element of P. Denote by $\mathcal{I}(P)$ the family of maximal intervals of P. The hypergraph $\mathcal{H}(P) = (P, \mathcal{I}(P))$, briefly denoted $\mathcal{H} = (P, \mathcal{I})$, whose vertices are the elements of P and whose edges are the maximal intervals of P is said to be the *order-interval hypergraph of* P. The *line-graph* $L(\mathcal{H})$ of $\mathcal{H}$ is a graph whose vertices are points $e_1, \dots, e_m$ representing the edges $I_1, \dots, I_m$ of $\mathcal{H}$, the vertices e_i, e_j being adjacent iff $I_i \cap I_j \neq \emptyset$. The dual $\mathcal{H}^*$ of the order-interval hypergraph $\mathcal{H}$ is a hypergraph whose vertices $e_1, \dots, e_m$ correspond to intervals of P and whose edges are $X_i = \{e_j : x_i \in I_j\}$.

Let α, ν, τ and ρ be the independence, matching, edge-covering and vertex-covering number of a hypergraph $\mathcal{H}$, respectively. $\mathcal{H}$ has the König property if $\nu(\mathcal{H}) = \tau(\mathcal{H})$ and it has the dual König property if $\nu(\mathcal{H}^*) = \tau(\mathcal{H}^*)$, i.e $\alpha(\mathcal{H}) = \rho(\mathcal{H})$ since $\alpha(\mathcal{H}) = \nu(\mathcal{H}^*)$ and $\rho(\mathcal{H}) = \tau(\mathcal{H}^*)$. This class of hypergraphs has been studied intensively in the past and one finds interesting results from an algorithmic point of view as well as min-max relations [2]-[6], [9].

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A poset P is said to be a series-parallel poset, if it can be constructed from singletons using only two operations: disjoint sum and linear sum. It may be characterized by the fact that it does not contain the poset N of Figure 3 as an induced subposet [13], [14].

Let P be a finite poset. The graph $G_P = (P, E_P)$, with $xy \in E_P$ if $x < y$ or $y < x$ is the comparability graph of the poset P. $G = (V, E)$ is a comparability graph if there is a poset P such that $G \sim G_P$.

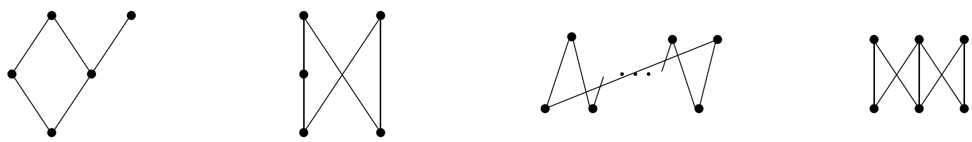
It is known that the cographs, i.e. graphs without an induced path of length 4, are comparability graphs of series-parallel posets [7]. The cographs belong to the class of distance-hereditary graphs, which has been studied in graph theory [7]. A possible definition of a distance-hereditary graph is as follows: G is a distance-hereditary graph iff G has no induced gem, house, hole (cycle of length at least 4) and domino (see Figure 1).

gem house hole domino

Figure 1

We investigate a class of posets that contains the series-parallel posets and whose members have comparability graphs which are distance-hereditary graphs or generalizations of them.

A poset P is in the class $\mathcal{Q}$ (resp. $\mathcal{Q}'$) if it has no induced subposet isomorphic to P_1, P_2, P_3 (resp. P_1, P_2, P_3, P_4) of Figure 2 and their duals, where P_3 has n vertices, $n \geq 6$. Obviously the class $\mathcal{Q}'$ is included in $\mathcal{Q}$. We prove that if P is in $\mathcal{Q}$, then $\mathcal{H}(P)$ has the dual König property.



P_1 P_2 P_3 P_4

Figure 2

We characterize the comparability graphs of the class of posets in $\mathcal{Q}'$ in terms of four forbidden subgraphs.

Proposition 1 *Let G be the comparability graph of the poset P. Then G contains no induced gem, house, domino and even hole if and only if $P \in \mathcal{Q}'$.*

Proof: We prove this result in four steps.

Step 1. The graph G contains no induced gem if and only if P contains neither P_1 nor P_1^* as an induced subposet. Indeed, assume that P has an induced P_1 (resp. P_1^*) and let x, y, z, t, u be the elements of P_1 such that $x < y < z > t > x$ and $t < u$ (resp. $t > u$). We immediately deduce a gem with edges xy, xz, xt, xu, yz, zt and tu (resp. zy, zx, zt, zu, yx, xt and tu) of G. Conversely, suppose that the graph G has an induced gem whose edges are xy, xz, xt, xu, yz, zt and tu. The subgraph of G induced by $\{x, y, z\}$ (resp. $\{x, z, t\}$) is a triangle, hence x, y, z (resp. x, z, t) form a chain of P. As $yt \notin E$, we obtain only six possibilities: $z < y < x > t > z$ or $x < y < z > t > x$ or $y, t < z < x$ or $t, y < x < z$ or $x < z < y, t$ or $z < x < y, t$. In virtue of the existence of the triangle induced by $\{x, u, t\}$, we infer that the first case gives $z < y < x > t > u, z$, the second $z > y > x < t < z, u$, the third $t < u < x > z > t, y$, the fifth $t > u > x < z < y, t$, without another comparability relation, and the fourth and sixth lead to a contradiction. Hence, we have obtained in each case either P_1 or P_1^*.

Step 2. The graph G contains no induced house if and only if P contains no P_2 as an induced subposet. Indeed, assume that P has an induced P_2 and let x, y, z, t, u be the elements of P_2 such that $x < y < z > t < u > x$. We immediately deduce a house with edges xy, yz, xz, zt, tu and ux of G. Conversely, suppose that G has an induced house whose edges are xy, yz, xz, zt, tu and ux. Since $xy \in E$, the elements x and y are comparable. First, assume that $x < y$. As $yz \in E$, we have $y < z$ or $z < y$. In fact $z < y$ leads to a contradiction. To see this, note that if $z < y$ holds, then $z < x < y$ or $x < z < y$. In the first case, from $ux \in E$, we deduce $u > x$, i.e. $u > z$, or $u < x$, i.e. $u < y$, both impossible since uz and uy are not edges of E. In the second case, $zt \in E$ implies $z < t$, i.e., $x < t$ or $z > t$, i.e. $t < y$, both impossible since xt and ty are not edges of E. Hence $z < y$ is impossible. From $tz \in E$ and $yt \notin E$, we obtain $x < y < z > t$ and these are the only comparability relations. As $tu \in E$ and $uz \notin E$, we deduce $t < u$. Finally, the only possibility for the relation between x and u is $x < u$. Hence, P_2 is obtained as an induced subposet. Adopting the same argument for $y < x$, we obtain P_2 as an induced subposet with the ordering $y < x < z > t < u > y$.

Step 3. It is easy to see that G contains no induced even hole if and only if P does not contain a P_3 as an induced subposet.

Step 4. The graph G contains no induced domino if and only if P contains no P_4 as an induced subposet. Indeed, assume that P has an induced P_4 and let x, y, z, t, u, v be elements of P_4 such that $x < t, u$ and $y < t, u, v$ and $z < u, v$. We immediately deduce a domino with edges xt, ty, yv, vz, zu, ux and uy of G. Conversely, suppose that G has an induced domino whose edges are xt, ty, yv, vz, zu, ux and uy. Hence, $ux, uy, uz \in E$ and $xy, yz, xz \notin E$ lead to $x, y, z < u$ or $u < x, y, z$ with $x \parallel y$, $y \parallel z$ and $x \parallel z$. We consider only the first possibility because the other may be settled by duality. From $yt \in E$ (resp. $yv \in E$) and $ut \notin E$ (resp. $uv \notin E$), we obtain $y < t$ (resp. $y < v$). For the remaining edges xt and zv, we have only the possibilities $x < t$ and $z < v$. Obviously, there are no other comparability relations between these elements. □

By Proposition 1, the comparability graph of a poset in $\mathcal{Q}'$ is a distance-hereditary graph, because the comparability graph of any poset cannot contain an odd hole: Each transitive orientation of an odd hole contains two consecutive arcs xy and yz which imply the chord xz.

In order to prove the dual König property of $\mathcal{H}(P)$ when P is in the class $\mathcal{Q}$, let us introduce two observations. We recall that the vertices of the line-graph $L(\mathcal{H}^*(P))$ are the points of P and two vertices are adjacent iff they belong to the same interval of P.

Observation 1 *Assume that P has no induced subposet isomorphic to P_1 and P_1^*. Let $u, v, w \in P$ with $u \parallel v$. If there exist two intervals I and I' such that $u, v \in I$ and $v, w \in I'$, then $u \in I'$.*

Proof: Let $I = [p, q]$ and $I' = [p', q']$. If $u \notin I'$, then $u \not< q'$ or $p' \not< u$. In the first case, the poset induced by $\{p, u, v, q, q'\}$ and P_1 are isomorphic. In the second case, the poset induced by $\{p, p', u, v, q\}$ and P_1^* are isomorphic, both impossible. □

By Observation 1, one can say that the existence of two edges uv and vw of the line-graph $L(\mathcal{H}^*(P))$ with the above mentioned properties enables us to affirm that uw is an edge, too.

Observation 2 *Assume that P has no induced subposet isomorphic to P_1, P_1^* and P_3. Let the 'zig zag' $u_1 < u_2 > u_3 < \cdots > u_{i-1} < u_i$, be given by i elements of P, linking u_1 to u_i, where i is even, $i \geq 6$. If u_1 and u_i belong to the same interval of P, then there exists at least another comparability relation between $u_1, \ldots, u_i$, different from $u_1 < u_i$ and $u_i < u_1$.*

Proof: If $u_1 > u_i$, then $u_1 > u_{i-1}$. If $u_1 \parallel u_i$, then from Observation 1, $u_i, u_2 \in I_1$, where I_1 is the interval containing u_1 and u_2. If $u_1 < u_i$, then there exists at least another comparability relation between $u_1, \ldots, u_i$, different from $u_1 < u_i$ and $u_i < u_1$, because otherwise the poset induced by $\{u_1, u_2, \ldots, u_i\}$ and P_3 would be isomorphic. □

Theorem 1 *Let $\mathcal{H}(P)$ be the order-interval hypergraph of a poset P of the class $\mathcal{Q}$. Then the line-graph $L(\mathcal{H}^*(P))$ is perfect.*

Proof: It is enough to verify that the line-graph $L(\mathcal{H}^*(P))$ is a Meyniel graph, i.e. each cycle of odd length at least 5 has at least two chords. Meyniel [11] proved the perfectness of Meyniel graphs.

Let $\mathcal{C} = (a_1, \ldots, a_k)$ be a cycle of odd length k, $k \geq 5$. Let us denote by $I_i = [p_i, q_i]$ the interval of P containing both a_i and a_{i+1} and by $I = [p, q]$ the interval of P containing both a_1 and a_k.

Case 1. $a_1 \parallel a_2$. From Observation 1, we have $a_1 a_3 \in I_2$ and $a_2 a_k \in I$.

Case 2. $a_1 < a_2$. We distinguish three subcases:

Case 2.1. $a_2 \parallel a_3$. From Observation 1, we have $a_1, a_3 \in I_1$ and $a_2, a_4 \in I_3$.

Case 2.2. $a_2 < a_3$. We immediately deduce the existence of the chord $a_1 a_3$ of $\mathcal{C}$. Let us determine another chord.

Case 2.2.1. $a_3 < a_4$ or $a_3 \parallel a_4$. Then $a_2 a_4$ is a chord of $\mathcal{C}$. Indeed, $a_3 < a_4$ implies $a_2 < a_4$ and from Observation 1, $a_3 \parallel a_4$ leads to $a_2, a_4 \in I_2$.

Case 2.2.2. $a_3 > a_4$. Then $a_3 a_5$ is a chord of $\mathcal{C}$ if $a_4 > a_5$ or $a_4 \parallel a_5$. Indeed, $a_4 > a_5$ implies $a_3 > a_5$ and from Observation 1, $a_4 \parallel a_5$ leads to $a_3, a_5 \in I_3$.

Now let $a_4 < a_5$. In the case $k = 5$, we have three possibilities: If $a_1 > a_5$ or $a_1 \parallel a_5$, then $a_2 a_5$ is a chord of $\mathcal{C}$. Indeed, $a_1 > a_5$ implies $a_2 > a_5$ and from Observation 1, $a_1 \parallel a_5$ leads to $a_2, a_5 \in I_1$. If $a_1 < a_5$, then we must have another comparability relation between the elements a_1, a_2, a_3, a_4, a_5, i.e. the existence of a new chord, because otherwise the poset induced by $\{a_1, a_2, a_3, a_4, a_5\}$ and P_2 would be isomorphic. In the case $k > 5$, consider the 'zig zag' $a_1 < a_3 > a_4 < a_5 > \cdots > a_{i-1} < a_i$ linking a_1 to a_i where i is a maximum odd integer, $5 \leq i \leq k$. If $i = k$, i.e a_1, a_i are in the same interval of P, we use Observation 2 to affirm the existence of the second chord. If $i < k$, we have again three possibilities:

If $a_{i+1} > a_i$ or $a_{i+1} \parallel a_i$, then $a_{i-1} a_{i+1}$ is a chord of $\mathcal{C}$. Indeed, $a_i < a_{i+1}$ implies $a_{i+1} > a_{i-1}$ and from Observation 1, $a_i \parallel a_{i+1}$ leads to $a_{i-1}, a_{i+1} \in I_{i-1}$. If $a_{i+1} < a_i$, the cases $a_{i+1} > a_{i+2}$ and $a_{i+1} \parallel a_{i+2}$ give a new chord $a_i a_{i+2}$ since $a_{i+2} < a_{i+1}$ implies $a_{i+2} < a_i$ and from Observation 1, $a_{i+1} \parallel a_{i+2}$ implies $a_i, a_{i+2} \in I_i$.

Case 2.3. $a_2 > a_3$. We distinguish three subcases:

Case 2.3.1. $a_3 \parallel a_4$. From Observation 1, $a_2, a_4 \in I_2$ and $a_3 a_5 \in I_4$.

Case 2.3.2. $a_3 > a_4$. Then $a_2 a_4$ is a chord of $\mathcal{C}$ since $a_4 < a_3 < a_2$. Now, if $a_4 > a_5$ or $a_4 \parallel a_5$, we deduce the chord $a_3 a_5$ since $a_4 > a_5$ implies $a_3 > a_5$ and from Observation 1, $a_4 \parallel a_5$ implies $a_3, a_5 \in I_3$. If $a_4 < a_5$, then the corresponding part of this case in Case 2.2.2. remains valid here by considering the 'zig zag' $a_1 < a_2 > a_4 < a_5 > \cdots > a_{i-1} < a_i$.

Case 2.3.3. $a_3 < a_4$. If $a_4 < a_5$, then $a_3 < a_5$, i.e. $a_3 a_5$ is a chord of $\mathcal{C}$. For obtaining the second chord, we continue as in Case 2.2.2 (from the same situation $a_4 < a_5$). Here the 'zig zag' is $a_1 < a_2 > a_3 < a_5 > a_6 < \cdots > a_{i-1} < a_i$. If $a_4 \parallel a_5$, then from Observation 1, we have on the one hand $a_3, a_5 \in I_3$. On the other hand $a_1, a_4 \in I$ if $k = 5$ and $a_4, a_6 \in I_5$

otherwise. If $a_4 > a_5$ and $k = 5$, then either $a_5 < a_1$ (resp. $a_1 < a_5$) or $a_1 \parallel a_5$. If $a_5 < a_1$ (resp. $a_1 < a_5$), not only $a_5 < a_2$ (resp. $a_1 < a_4$), i.e. a_2a_5 (resp. a_1a_4) is a chord of $\mathcal{C}$ but again, it must exist another comparability relation between elements $a_1, \ldots, a_5$ because otherwise, the poset induced by these elements and P_2 would be isomorphic. If $a_1 \parallel a_5$, we have by Observation 1, $a_2, a_5 \in I_1$ and $a_1, a_4 \in I_4$, hence a_2a_5 and a_1a_4 are chords of $\mathcal{C}$.

If $a_4 > a_5$ and $k > 5$, consider the 'zig zag' $a_1 < a_2 > a_3 < \cdots < a_{i-1} > a_i$, where i is a maximum odd integer, $5 \leq i < k$.
If $i = k$, we have either, $a_1 > a_i$ (resp. $a_1 < a_i$) or $a_1 \parallel a_i$. If $a_1 > a_i$ (resp. $a_1 < a_i$), a_2a_i (resp. a_1a_{i-1}) is a chord of $\mathcal{C}$. Moreover there exists another comparability relation between elements $a_2, \ldots, a_i$ (resp. $a_1, \ldots, a_{i-1}$) because otherwise the poset induced by these elements and P_3 would be isomorphic. If $a_1 \parallel a_i$, by Observation 1, $a_1, a_{i-1} \in I_{i-1}$ and $a_2, a_i \in I_1$.
If $i < k$, we have three subcases:

Case 2.3.3.1. $a_{i+1} \parallel a_i$. Then from Observation 1, $a_{i-1}, a_{i+1} \in I_{i-1}$ and $a_i, a_{i+2} \in I_{i+1}$.

Case 2.3.3.2. $a_{i+1} < a_i$. We immediately deduce $a_{i+1} < a_{i-1}$, i.e. the chord $a_{i-1}a_{i+1}$ of $\mathcal{C}$.

If $a_{i+1} > a_{i+2}$, then a_ia_{i+2} is a chord of $\mathcal{C}$. If $a_{i+1} \parallel a_{i+2}$, then from Observation 1, $a_ia_{i+2} \in I_i$. If $a_{i+1} < a_{i+2}$, we continue as in Case 2.2.2. with the zig zag' $a_1 < a_2 > \cdots < a_{i-1} > a_{i+1} < a_{i+2}$.

Case 2.3.3.3. $a_{i+1} > a_i$. If $a_{i+1} < a_{i+2}$, then $a_i < a_{i+2}$ and hence a_ia_{i+2} is a chord of $\mathcal{C}$. In the case $k = i + 2$, we have either $a_1 < a_{i+2}$ which leads to the existence of another comparability relation between the elements $a_1, \ldots, a_i, a_{i+2}$, i.e. a new chord, since otherwise the poset induced by these elements and P_3 would be isomorphic, or $a_1 > a_{i+2}$ or $a_1 \parallel a_{i+2}$. These last possibilities give the chord a_1a_{i+1} of $\mathcal{C}$ because $a_{i+2} < a_1$ implies $a_{i+1} < a_1$ and from Observation 1, $a_1 \parallel a_{i+2}$ implies $a_1, a_{i+1} \in I_{i+1}$.
In the case $i + 2 < k$, we consider the 'zig zag' $a_1 < a_2 > \cdots < a_{i-1} > a_i < a_{i+2}$ and we continue as in Case 2.2.2. by substituting the elements $a_3, \ldots, a_{i-2}, a_{i-1}, a_i$ by $a_2, \ldots, a_{i-1}, a_i, a_{i+2}$, respectively.
If $a_{i+1} \parallel a_{i+2}$, then from Observation 1, $a_i, a_{i+2} \in I_i$ and $a_1, a_{i+1} \in I$ (resp. $a_{i+1}, a_{i+3} \in I_{i+2}$) if $k = i + 2$ (resp. $k > i + 2$).

Case 3. $a_2 < a_1$. By duality, this case is similar to Case 2.

Finally, we have obtained in each case at least two chords of $\mathcal{C}$ and the proof is complete. □

Let $\mathcal{H} = (E_1, \ldots, E_m)$ be a hypergraph. We say that $\mathcal{H}$ has the *Helly property* or is a *Helly hypergraph* if every intersecting family of $\mathcal{H}$ is a star, i.e. for $J \subset \{1, \ldots, m\}$, $E_i \cap E_j \neq \emptyset$, for $i, j \in J$, implies $\cap_{j \in J} E_j \neq \emptyset$. A good characterization of a Helly hypergraph, due to Berge and Duchet [1], is given by the following property:

For any three vertices a_1, a_2, a_3 the family of edges containing at least two of the vertices a_i

has a non-empty intersection.

Theorem 2 *Let $\mathcal{H}(P)$ be the order-interval hypergraph of a poset P which has no induced subposet isomorphic to P_1 and P_1^*. Then $\mathcal{H}^*(P)$ is a Helly hypergraph.*

Proof: In the class of order-interval hypergraphs of posets, $\mathcal{H}^*(P)$ is a Helly hypergraph if and only if $\mathcal{H}(P)$ is a Helly hypergraph [5]. Consequently, we can verify this property for the hypergraph $\mathcal{H}(P)$.

Let $\mathcal{I} = \{I_1, \ldots, I_m\}$ be the family of maximal intervals of P. We suppose that there exist three vertices a_1, a_2, a_3 of P such that $\cap_{j \in J} I_j = \emptyset$ where $J = \{j : |I_j \cap \{a_1, a_2, a_3\}| \geq 2\}$. Hence, $|J| \geq 3$ and there exists three edges, say w.l.o.g $I_1 = [p_1, q_1]$, $I_2 = [p_2, q_2]$, $I_3 = [p_3, q_3]$, such that:

$$\begin{aligned} a_2, a_3 \in I_1 \quad &\textit{and} \quad a_1 \notin I_1 \\ a_1, a_3 \in I_2 \quad &\textit{and} \quad a_2 \notin I_2 \\ a_1, a_2 \in I_3 \quad &\textit{and} \quad a_3 \notin I_3 \end{aligned}$$

From Observation 1, we have $a_1 \in I_1$ if $a_1 \parallel a_2$, and $a_2 \in I_2$ if $a_1 < a_2$ and $a_2 \parallel a_3$. If $a_1 < a_2$ and $a_2 < a_3$, we have immediately $a_2 \in I_2$. Again, we obtain $a_3 \in I_3$, if $a_1 < a_2$ and $a_3 < a_2$. Indeed, we must have $a_1 \parallel a_3$ because $a_1 < a_3$ (resp. $a_3 < a_1$) implies $p_3 < a_1 < a_3 < a_2 < q_3$ (resp. $p_1 < a_3 < a_1 < a_2 < q_1$), i.e. $a_3 \in I_3$ (resp. $a_1 \in I_1$). Moreover, $p_2 \neq p_3$, because otherwise $p_3 = p_2 < a_3 < a_2 < q_3$ and hence, $a_3 \in I_3$. Consequently, the poset induced by $\{p_2, p_3, a_1, a_3, a_2\}$ and P_1^* are isomorphic. By duality, the remaining case, namely $a_2 < a_1$, leads to a contradiction as well. □

A hypergraph $\mathcal{H}$ is said to be normal if every partial hypergraph $\mathcal{H}'$ has the coloured edge property, i.e. it is possible to colour the edges of $\mathcal{H}'$ with $\triangle(\mathcal{H}')$ colours, where $\triangle(\mathcal{H}')$ represents the maximum degree of $\mathcal{H}'$. Several sufficient conditions exist for a hypergraph to have the König property [1]. One of them is its normality. A hypergraph $\mathcal{H}$ is normal iff it satisfies the Helly property and the line-graph $L(\mathcal{H})$ is a perfect graph. This characterization enables us to derive the following corollary.

Corollary 3 *Let $\mathcal{H}(P)$ be the order-interval hypergraph of a poset P of the class $\mathcal{Q}$. Then every subhypergraph of $\mathcal{H}(P)$ has the dual König property.*

Proof: By Theorem 1 and Theorem 2, $\mathcal{H}^*(P)$ is normal and consequently every partial hypergraph is again normal. As the dual of a partial hypergraph of $\mathcal{H}^*(P)$ is a subhypergraph of $\mathcal{H}(P)$, we deduce that every subhypergraph of $\mathcal{H}(P)$ has the dual König property. □

2 N-free posets

Another natural and interesting generalization of series-parallel posets is the class of N-free poset. A poset is called N-free iff its Hasse-diagram does not contain the N from

Figure 3 as an induced subgraph [12], i.e. if there do not exist vertices $v_1, \ldots, v_4$ such that $v_1 \prec v_3 \succ v_2 \prec v_4$ and $v_1 \parallel v_4$.

There is a characterization of series-parallel posets within the class of N-free posets [8]. It states that a poset P is a series-parallel iff P is N-free and does not contain the poset N' of Figure 3 as an induced subposet.

N N'

Figure 3

Unfortunately, if the poset P is N-free, the König property is not satisfied in general. The poset of Figure 4, gives a counterexample since $\nu(\mathcal{H}(P_1)) = 1$ and $\tau(\mathcal{H}(P_1)) = 2$. Moreover, $\mathcal{H}^*(P)$ is not normal. To see this, consider the poset P_2 of Figure 4. The line-graph $L(\mathcal{H}^*(P))$ contains an induced odd cycle C_5 given by the vertices $\{2, 3, 4, 12, 13\}$ and hence $L(\mathcal{H}^*(P))$ is not perfect.

P_1 P_2

Figure 4

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