# Pairwise Balanced Designs whose Block Size Set Contains Seven and Thirteen

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**Abstract**

In this paper, we investigate the PBD-closure of sets $K$ with $\{7, 13\} \subseteq K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$. In particular, we show that $v \equiv 1 \bmod 6$, $v \geq 98689$ implies $v \in B(\{7, 13\})$. As a preliminary result, many new 13-GDDs of type $13^q$ and resolvable BIBD with block size 6 or 12 are also constructed. Furthermore, we show some elements to be not essential in a Wilson bases for the PBD-closed set $\{v : v \equiv 1 \bmod 6, v \geq 7\}$.

Last compiled: June 28, 2005

# 1 Introduction

Let $K$ be a set of positive integers. Then a *pairwise balanced design* PBD$(v, K)$ of order $v$ with block sizes from $K$ is a pair $(V, \mathcal{B})$, where $V$ is a finite set (the *point set*) of cardinality $v$ and $\mathcal{B}$ is a family of subsets (called *blocks*) of $V$ which satisfy the following properties:

(i) every pair of distinct elements of $V$ occurs in exactly one block of $\mathcal{B}$;

(ii) if $B \in \mathcal{B}$, then $|B| \in K$.

In a sequence of three papers R.M. Wilson [25, 26, 28] developed a theory of PBD-closed sets and the notation of PBD-closure. A set $S$ of positive integers is said to be *PBD-closed* if the existence of a PBD$(v, S)$ $(v > 0)$ implies that $v$ belongs to $S$. Let $K$ be a set of positive integers and let $B(K) = \{v > 0 : \exists \text{ PBD}(v, K)\}$. Then $B(K)$ is a PBD-closed set called the *PBD-closure* of $K$. According to Wilson's theory there exists a constant $c_0(K)$ such that designs PBD$(v, K)$ exist for all $v \geq c_0(K)$ which satisfy the congruences $(v-1) \equiv 0 \bmod \alpha(K)$ and $v(v-1) \equiv 0 \bmod \beta(K)$, where $\alpha(K) = \gcd\{k-1 : k \in K\}$ and $\beta(K) = \gcd\{k(k-1) : k \in K\}$. Concerning the structure of PBD-closed sets Wilson also showed that if $S$ is a PBD-closed set, then $S$ is *eventually periodic* with period $\beta(S)$; that is, there exists a constant $c_0(S)$ such that for every $k \in S$, $\{v : v \geq c_0(S), v \equiv k \bmod \beta(S)\} \subseteq S$.

The theory of PBD-closed sets is a powerful tool for investigations of combinatorial structures, a finite number of known examples of a certain set of objects can establish the existence of the entire set of these objects, for examples, see [22, p. 203] and [27].

Unfortunately, the constant $c_0(K)$ is not known in general. Although Wilson's proof is somehow constructive, the estimate of the constant is very large. Therefore, one attempts to determine $B(K)$ for given $K$ as accurately as possible, for a survey, see, for example, [22, Tables 3.17, 3.18].

In this paper, we investigate the PBD-closure of sets $K$ with $\{7, 13\} \subseteq K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$. Our motivation to consider these sets $K$ comes from a problem on pan-orientable block designs. In [19] it is shown that the set of values $v$ which admit a pan-orientable block design $(v, 4, 2)$ is a PBD-closed set containing 7 and 13 and is a subset of $N_{1,6} = \{v : v \geq 6, v \equiv 1 \bmod 6\}$. We are able to give in Section 5 an upper bound on $c_0(\{7, 13\})$: $c_0(\{7, 13\}) \leq 98689$. More precise upper bounds are obtained when looking at the residues modulo 42. In Table 1 we present the largest possible exception in each residue class $6t + 1$ modulo 42, $0 \leq t < 7$. Note that the largest exceptions in the residue classes $1, 7$ or 13 modulo 42 are small

| $6t+1$ modulo 42 | 1 | 7 | 13 | 19 | 25 | 31 | 37 |
|---|---|---|---|---|---|---|---|
| Largest exception | 2605 | 1645 | 14293 | 82549 | 98683 | 91507 | 88447 |

Table 1: Largest possible exception in each residue class $6t+1$ modulo 42

compared to largest exceptions for $19, 25, 31$ or $37$ modulo 42. Therefore, it seems opportune to include in $K$ the smallest values from each fibre modulo 42 in order to eliminate numbers that are in the list of exceptions for $B(\{7,13\})$. So, we study in Section 6 the PBD-closure of sets $K$ where $\{7,13\} \subset K \subseteq \{7,13,19,25,31,37,43\}$.

The constructions we used to establish these results are partially taken from Mullin and Stinson [23] which determined the PBD-closure of $P_{1,6}$, where $P_{1,6}$ is defined to be the set of prime powers congruent 1 modulo 6. They showed that $B(P_{1,6}) \cup Q = N_{1,6}$, where $Q$ is a set of 31 (possible) exceptions. Subsequently, their result was improved by Greig [17] who removed 9 of the values in $Q$. In Section 4 a number of new constructions, which are based on Wilsons Fundamental Construction, are introduced. As ingredients for these constructions we will construct many new 13-GDDs of type $13^q$ and (resolvable) BIBD with block size $6, 12$ or $13$.

As a consequence of Wilson's existence theory it follows that if $S$ is a PBD-closed set, then there exists a finite subset $J \subseteq S$ such that $S = B(J)$. Such a set $J$ is said to be a *finite basis* or *Wilson basis* for the closed set $S$. An element $x \in J$ is called *essential* if and only if $x \notin B(J \setminus \{x\})$. Mullin [21] determined a first Wilson basis for the set $N_{1,6} = B(\{7,$ 13, 19, 25, 31, 37, 43, 55, 61, 67, 73, 79, 97, 103, 109, 115, 121, 127, 139, 145, 157, 163, 181, 193, 199, 205, 211, 223, 229, 235, 241, 253, 265, 271, 277, 283, 289, 295, 307, 313, 319, 331, 349, 355, 361, 367, 373, 379, 391, 397, 409, 415, 421, 439, 445, 451, 457, 487, 493, 499, 643, 649, 655, 661, 667, 685, 691, 697, 709, 727, 733, 739, 745, 751, 781, 787, 811, 1063, 1069, 1231, 1237, 1243, 1249, 1255, 1315, 1321, 1327, 1543, 1549, 1567, 1579 1585, 1783, 1789, 1795, 1801, 1819, 1831$\}$). Independently, Du [14] and Greig [17] removed the elements 223, 253, 295, 307, 361, 367, 379, 421, 439, 655, 727, 1231, 1237, 1243, 1249, 1255, 1543, 1549, 1585, 1783, 1789, 1795, 1801, 1819 and 1831 from the finite basis for $N_{1,6}$. We will improve this result in Section 4 by showing that 4 further values 1063, 1069, 1567 and 1579 are not essential.

## 2 Preliminaries

In this section, we give some definition and notations as well as some preliminary results which will be used in the sequel. We refer the reader to [9] and [12] for undefined terms as well as a general overview of design theory.

Fundamental to our constructions are a number of designs which we define now. A *group-divisible design* (GDD) is a triple $(V, \mathcal{G}, \mathcal{B})$ where $V$ is a set of *points*, $\mathcal{G}$ is a partition of $V$ into *groups* and $\mathcal{B}$ is a collection of subsets of $V$ (called *blocks*) such that any pair of distinct points in $V$ occurs together either in some group or in exactly one block, but not both. A $K$-GDD of type $g_1^{t_1} g_2^{t_2} \dots g_r^{t_r}$ is a GDD in which each block has size from the set $K$ and in which there are $t_i$ groups of size $g_i, i = 1, 2, \dots, r$. We will denote a $\{k\}$-GDD as a $k$-GDD. Group divisible designs are useful as ingredients in Wilson's Fundamental Construction.

**Construction 2.1** *Let $(V, \mathcal{G}, \mathcal{B})$ be a GDD and let $w : V \to \mathbb{Z}^+ \cup \{0\}$ (w is called a* weight *function). Suppose that for each block $B \in \mathcal{B}$ there is a $K$-GDD of type $\{w(x) : x \in B\}$. Then there is a $K$-GDD of type $\{\sum_{x \in G_i} w(x) : G_i \in \mathcal{G}\}$.*

A *transversal design* TD$(k, n)$ is a $k$-GDD of type $n^k$. It is well-known that the existence of a TD $(k, n)$ is equivalent to that of $k - 2$ mutually orthogonal Latin squares (MOLS) of side $n$. Also, it is well known that for all prime powers $q$ there is a TD$(q+1, q)$. Our source of TDs is the following result of MacNeish [20].

**Lemma 2.2** *If a TD$(k, m)$ and a TD$(k, n)$ exist, then a TD$(k, mn)$ exists.*

Moreover, we use the full knowledge of the updated MOLS table [4, 13, 6] which provides a list of lower bounds on the number of MOLS of all orders up to 10000. In particular, TD$(14, m)$ play an important role in one of our constructions. Thus, we mention the following result [4, Table 2.68].

**Lemma 2.3** *If $m \geq 6713$ and $m \equiv 0 \mod 7$, or if $m \geq 3567$ and $m$ is odd, or if $m \geq 7289$, then there is a TD$(14, m)$.*

We also require the notion of incomplete designs. An *incomplete pairwise balanced design* IPBD$(v, h, K)$ is a triple $(V, Y, \mathcal{B})$ where $V$ is a set of $v$ points, $Y$ is a subset of $V$ of size $h$ ($Y$ is called the *hole*), and $\mathcal{B}$ is a collection of subsets of $V$ (blocks) such that

(i) any pair of distinct elements of $V$ occur together either in the hole $Y$ or in exactly one block of $\mathcal{B}$, but not both;

(ii) if $B \in \mathcal{B}$, then $|B| \in K$.

Suppose $(V, Y, \mathcal{B})$ is an IPBD$(v, h, K)$ and $(Y, \mathcal{Y})$ is a PBD$(h, K)$. Then we say that $(V, \mathcal{B} \cup \mathcal{Y})$ is a PBD with a *flat* $Y$ of order $h$. Note that in any PBD any block can be considered to be a flat.

**Lemma 2.4** *Suppose there is an IPBD$(v, w, K)$ and an IPBD$(w, h, K)$. Then there is an IPBD$(v, h, K)$.*

**Proof.** Fill the hole $W$ of the IPBD$(v, w, K)$ with the blocks of the IPBD$(w, h, K)$ defined on the point set $W$ to obtain the desired design. □

An *incomplete transversal design* ITD$(k; n, m)$ is a quadruple $(V, Y, \mathcal{G}, \mathcal{B})$ where $V$ is a set of $kn$ points, $Y$ is a subset of $V$ of size $km$ ($Y$ is called the *hole*), $\mathcal{G}$ is a partition of $V$ into $k$ groups, each of size $n$, and $\mathcal{B}$ is a collection of $k$-subsets of $V$ (blocks) such that

(i) for each group $G_i \in \mathcal{G}$, $|G_i \cap Y| = m$,

(ii) for each group $G_i \in \mathcal{G}$ and each block $B_j \in \mathcal{B}$, $|G_i \cap B_j| = 1$, and

(iii) any pair of points from distinct groups occur together either in the hole $Y$ or in exactly one block, but not both.

Here, we list five criteria for determining the existence of ITDs which follow from results of Brouwer and van Rees [10].

**Lemma 2.5** *Suppose there exists a TD$(k, m)$, a TD$(k, m+1)$, a TD$(k+1, t)$, and $0 \le u \le t$. Then there exists an ITD$(k; mt + u, u)$.*

**Lemma 2.6** *Suppose there exists a TD$(k, m)$, a TD$(k, m+1)$, a TD$(k, m+2)$, a TD$(k+2, t)$, a TD$(k, u)$ and $0 \le u, v \le t$. Then there exists an ITD$(k; mt + u + v, v)$.*

**Lemma 2.7** *Suppose there exists a TD$(k, m)$, a TD$(k, m+1)$, a TD$(k, m+2)$, a TD$(k+u+1, t)$, a TD$(k+1, m+u)$ and $0 \le v \le t-1$. Then there exists an ITD$(k; mt + u + v, v)$.*

**Lemma 2.8** *Let $m > 1$ and suppose there exist a TD$(k, m)$, a TD$(k, m+1)$ and a TD$(k+u, t)$. Then there exists an ITD$(k; mt + u, m + u)$.*

**Lemma 2.9** *If there exists a TD$(k, m)$, then there exists an ITD$(k; m, 1)$.*

# 3 Known Designs

In this section, we collect together previously constructed designs for later use.

A PBD$(v, \{k\})$ is usually called a *balanced incomplete block design* BIBD$(v, k, 1)$. *Resolvable* BIBDs are designs that admit a partition of the block set into subsets of blocks that contain every point exactly once. These designs are denoted by RBIBD$(v, k, 1)$. RBIBDs will be used to construct IPBDs and PBDs. Adjoining a new point to each partition of an RBIBD$(v, k, 1)$ yields an IPBD$(v + (v-1)/(k-1), (v-1)/(k-1), \{k+1\})$ and, if $\{k+1, (v-1)/(k-1)\} \subset B(K)$, a PBD$(v + (v-1)/(k-1), K)$.

BIBD$(v, 7, 1)$ have been extensively studied for a long time. The following result is the culmination of the contributions of several authors, for a survey see, for example, [3].

**Lemma 3.1** *A BIBD$(v, 7, 1)$ exists if $v \equiv 1$ or 7 mod 42 except when $v = 43$ and possibly when $v = 42t + 1$ and $t \in \{2, 3, 5, 6, 12, 14, 17, 19, 22, 27, 33, 37, 39, 42, 47, 59, 62\}$ or $v = 42t + 7$ and $t \in \{3, 19, 34, 39\}$.*

Resolvable BIBDs can be constructed from block disjoint difference families by the Ray-Chaudhuri-Wilson construction, see [24]. Difference families with block size 6 (which can all be made block disjoint) have been investigated by Chen and Zhu [11].

**Lemma 3.2 ([11, 24, 18])** *Let $q = 30t + 1, q \neq 61$ be a prime power. Then there exists a BIBD$(q, 6, 1)$ obtained from a block disjoint difference family, an RBIBD$(6q, 6, 1)$ and an IPBD$(216t + 7, 36t + 1, \{7\})$.*

We also know the following finite set of RBIBD$(v, 6, 1)$s.

**Lemma 3.3 ([2, 16, 18])** *If $4 \leq t \leq 832$, $t$ is even and $6t + 1$ is a prime power, or if $6 \leq t \leq 296$, $t$ is even and $5t + 1 \not\equiv 1$ mod 50 is a prime power, or if $t \in \{5, 57\}$, then an RBIBD$(30t + 6, 6, 1)$ exists and hence an IPBD$(36t + 7, 6t + 1, \{7\})$.*

Furthermore, we mention the following group divisible designs.

**Lemma 3.4 ([8, 7, 3])** *There exist a 7-GDD of type $3^{15}$ and 7-GDDs of type $7^n$ for $n \equiv 1$ mod 6, $n \notin \{19, 115, 145, 205, 235\}$.*

**Lemma 3.5** *If $v \in B(\{7\})$, then there is a 7-GDD $6^{\frac{v-1}{6}}$.*

**Proof.** Delete just one arbitrary point in a BIBD$(v, 7, 1)$. □

**Lemma 3.6** *There exist a* $\{7,13\}$*-GDD* $6^{51}$ *and a* 7*-GDD* $6^{49}12^1$.

**Proof.** There is a PBD$(307,\{7,13\})$ with exactly one block of size 13, see for example [17, Lemma 13.11] or Lemma 4.19 ($u=7, v=49, w=7, a=6$). Delete a point not on the 13-block for the GDD of type $6^{51}$ or a point from the 13-block for the GDD of type $6^{49}12^1$. □

**Lemma 3.7 ([18])** *Let* $t$ *be an integer with* $t \notin \{2-6,\ 10-14,\ 16,\ 18-27,\ 29-32,\ 34,\ 35,\ 37-40,\ 42-48,\ 51-55,\ 59-62,\ 93-95,\ 98,\ 100-103,\ 107-111,\ 116-118,\ 138,\ 139,\ 146,\ 152,\ 154,\ 156-160,\ 163-167,\ 170-174,\ 177-181,\ 185-189,\ 191,\ 192,\ 194,\ 195,\ 199,\ 200,\ 201,\ 207-209,\ 215,\ 216,\ 219,\ 221,\ 228-230,\ 269,\ 270,\ 272,\ 275-278,\ 283,\ 285,\ 286,\ 326,\ 334,\ 339,\ 342\}$. *Then there is a* 7*-GDD of type* $42^t$. *If* $\{7,43\} \subset K$, *then there is a PBD*$(42t+1, K)$.

Some useful group divisible designs arise from transversal designs. A proof of the following lemma is given in [23, Lemmas 2.1, 2.2].

**Lemma 3.8** *If there is a TD*$(k,n)$, *then there is a* $\{k,n\}$*-GDD of type* $(k-1)^n(n-1)^1$. *If there is a TD*$(k,n-1)$, *then there is a* $\{k,n\}$*-GDD of type* $(k-1)^{n-1}(n-1)^1$.

**Corollary 3.9** *There exist a* $\{7,13\}$*-GDD* $6^{13}12^1$ *and a* $\{7,13\}$*-GDD* $6^{12}12^1$. *There exist a* 7*-GDD* $6^{49}48^1$ *and a* 7*-GDD* $6^{48}48^1$.

**Proof.** The first two GDDs are obtained from a TD$(7,13)$ and a TD$(7,12)$, respectively. The next two GDDs are obtained from a TD$(7,49)$ and a TD$(7,48)$, respectively. Note that blocks of size 49 are replaced by blocks of a BIBD$(49,7,1)$. □

The following is a result of Greig [17, Thm. 8.4] which is based on a construction of Brouwer using Baer subplanes.

**Lemma 3.10** *If* $q$ *is a prime power, and* $0 < t < q^2-q+1$, *then it follows that* $t(q^2+q+1)+q^2-q+1-t \in B(\{t+1, q+t, (q^2-q+1-t)^*\})$ *and that there exists an IPBD*$(t(q^2+q+1)+q^2-q+1-t, q^2-q+1-t, \{t+1, q+t\})$.

# 4 Constructions

From GDDs, we construct IPBDs by filling in the groups by appropriate ingredient IPBDs.

**Lemma 4.1** *Suppose there is a $K$-GDD of type $g_1, g_2, \ldots, g_n$. If we have an IPBD$(g_i + f, f, K)$ for $2 \leq i \leq n$, and we have a PBD$(g_1 + f, K)$, then there is a PBD$(G + f, K)$ (where $G = \sum_{i=1}^{i=n} g_i$) containing a flat of order $g_1 + f$.*

In the case our GDD is a TD, we get the following specialization of Lemma 4.1.

**Lemma 4.2** *Suppose there is a TD$(k, n)$. If $\{k, n\} \subset B(K)$, then there is a PBD$(kn, K)$ containing flats of order $k$ and $n$. If $\{k, n+1\} \subset B(K)$, then there is a PBD$(kn+1, K)$ containing flats of order $k$ and $n+1$.*

The second construction is a simple application of Wilson's Fundamental Construction.

**Lemma 4.3** *Suppose there is a TD$(t, m)$ and a $k$-GDD of type $a^t$. If $\{k, am\} \subset B(K)$, then there is a PBD$(atm, K)$ containing flats of order $k$ and $am$. If $\{k, am+1\} \subset B(K)$, then there is a PBD$(atm+1, K)$ containing flats of order $k$ and $am+1$.*

**Proof.** Take the TD$(t, m)$, a $t$-GDD $m^t$ as the master design in Wilson's Fundamental Construction, apply weight $a$ to all points and replace the blocks of size $t$ by the $k$-GDD of type $a^t$. In the first case use a PBD$(k, K)$ and a PBD$(am, K)$ to replace the blocks of size $k$ and to fill the groups of size $am$. In the second case adjoin a common point to all groups and fill these groups with a PBD$(am + 1, K)$. $\square$

The next constructions (Lemmas 4.4, 4.5, 4.6 and 4.10) are taken from [23]. They are applications of Wilson's Fundamental Construction together with filling in the groups by appropriate designs.

**Lemma 4.4** *Suppose $\{k, n, m(k-1)+1, m(n-1)+1, t(k-1)+1\} \subset B(K)$, a TD$(k, n)$, a TD$(k, n-1)$, and a TD$(n+1, m)$ exists, and $0 \leq t \leq m$. Then there is a PBD$((n-1)mk + t(k-1) + 1, K)$ containing flats of order $k, n, m(k-1)+1, m(n-1)+1$ and $t(k-1)+1$.*

**Corollary 4.5** *Suppose there is a TD$(14, m)$ and $0 \leq t \leq m$. If $\{7, 13, 6m+1, 12m+1, 6t+1\} \subset B(K)$, then there is a PBD$(84m+6t+1, K)$ containing flats of order $7, 13, 6m+1, 12m+1$ and $6t+1$.*

**Lemma 4.6** *Suppose there exist $K$-GDDs of group types $1^{n+1}$ and $1^n c^1$. If there is a TD$(n+1, m)$ and $0 \leq t \leq m$, then there exists a $K$-GDD of group type $m^n(t(c-1)+m)^1$.*

**Corollary 4.7** *Suppose $\{u, m, 6t+m\} \subset B(K)$ and a TD$(u, m)$ exists. If there is a PBD$(u+6, K)$ with a flat of order 7, and $0 \leq t \leq m$, then there exists a PBD$(mu+6t, K)$ with flats of order $m$ and $6t+m$.*

**Proof.** Take $n = u - 1$ and $c = 7$ in Lemma 4.6, noting that a PBD$(u, K)$ can be considered as a $K$-GDD $1^u$ and a PBD$(u + 6, K)$ containing a flat of order 7 can be considered as a $K$-GDD $1^{u-1}7^1$. Then fill in groups. □

**Corollary 4.8** *Suppose $u, m, 12t + m \in B(K)$ and a TD$(u, m)$ exists. If there is a PBD$(u + 12, K)$ with a flat of order 13, and $0 \leq t \leq m$, then there exists a PBD$(mu + 12t, K)$ with flats of order $m$ and $12t + m$.*

**Proof.** Take $n = u - 1$ and $c = 13$ in Lemma 4.6. □

**Lemma 4.9** *Suppose $\{u, m, 6t+m\} \subset B(K)$ and a TD$(u, m)$ exists. If there is a PBD$(u + 6, K)$ with a flat of order 7, a PBD$(u + 12, K)$ with a flat of order 13, and $0 \leq t \leq 2m$, then there exists a PBD$(mu + 6t, K)$ with flats of order $m$ and $6t + m$.*

**Proof.** Write $t = x + 2y$ with $x + y \leq m$. Give all points in the TD$(u, m)$ weight 1, except for $x$ and $y$ points in the last group, which get weight 7 and 13, respectively. Apply Wilson's Fundamental Construction using $K$-GDD $1^u$, $K$-GDD $1^{u-1}7^1$ and $K$-GDD $1^{u-1}13^1$ derived from given PBDs. Then fill in groups. □

**Lemma 4.10** *Suppose $\{n, m(n - 1) + 1\} \subset B(K)$, a TD$(n, n)$, and a TD$(n, m)$ exists. Then there is a PBD$((n^2 - 1)m + 1, K)$ containing flats of order $n$ and $m(n - 1) + 1$.*

In the following we prove a few further applications of Wilson's Fundamental Construction which are useful for our purposes.

**Lemma 4.11** *Let $\{7, 13\} \subset B(K)$, suppose there is a TD$(14, m)$ and $0 \leq t \leq m$. If there exist an IPBD$(6m + f, f, K)$, and*

*a) an IPBD$(12m + f, f, K)$ and $6t + f \in B(K)$, or*

*b) an IPBD$(6t + f, f, K)$ and $12m + f \in B(K)$, or*

*c) an IPBD$(12m + f, f, K)$, an IPBD$(6t + f, f, K)$ and $6m + f \in B(K)$,*

*then there is a PBD$(84m + 6t + f, K)$ containing flats of order $7, 13$ and a flat of order a) $6t + f$, or b) $12m + f$, or c) $6m + f$.*

**Proof.** Truncate all but $t$ points of the last group of the TD$(14, m)$ to obtain a $\{13, 14\}$-GDD $m^{13}t^1$. Giving weight 12 to all points of one untruncated group and weight 6 to all other points Wilson's Fundamental Construction yields a $\{7, 13\}$-GDD $(6m)^{12}(12m)^1(6t)^1$ where we use as ingredient GDDs a $\{7, 13\}$-GDD $6^{12}12^1$ and a $\{7, 13\}$-GDD $6^{13}12^1$ from Corollary 3.9. Now applying Lemma 4.1 completes the proof. □

**Lemma 4.12** *Let* $\{7, 13\} \subset B(K)$ *and suppose there is a* $TD(14, m)$*. If there exist an* $IPBD(6(m-1)+f, f, K)$

*a) and* $12m + f \in B(K)$*, or*

*b) an* $IPBD(12m + f, f, K)$ *and* $6(m-1) + f \in B(K)$*,*

*then there is a* $PBD(90m - 78 + f, K)$ *containing flats of order* $7, 13$ *and a flat of order a)* $12m + f$*, or b)* $6(m-1) + f$*.*

**Proof.** Truncate all but one point of a block of the TD$(14, m)$ to obtain a $\{13, 14\}$-GDD $m^1(m-1)^{13}$. Giving weight 12 to all points of the group of size $m$ and weight 6 to all other points Wilson's Fundamental Construction yields a $\{7, 13\}$-GDD $(12m)^1(6(m-1))^{13}$ where we use as ingredient GDDs a $\{7, 13\}$-GDD $6^{12}12^1$ and a $\{7, 13\}$-GDD $6^{13}12^1$ from Corollary 3.9. Again, applying Lemma 4.1 completes the proof. □

**Lemma 4.13** *Let* $\{7, 13\} \subset B(K)$*, suppose there is a* $TD(t+1, m)$*, assume* $\{6t+1, 6t+7\} \subset B(\{7\})$ *and* $0 \leq r \leq m$*. If there exist an* $IPBD(6m+f, f, K)$*,*

*a) and* $6r + f \in B(K)$*, or*

*b) an* $IPBD(6r + f, f, K)$ *and* $6m + f \in B(K)$*,*

*then there exists a* $PBD(6mt + 6r + f, K)$ *containing flats of order* $7$ *and a)* $6r + f$ *or b)* $6m + f$*.*

**Proof.** Truncate one group of the TD to size $r$ to obtain a $\{t, t+1\}$-GDD $m^t r^1$. Give weight 6 to all points and apply Wilson's Fundamental Construction with ingredient GDDs 7-GDD $6^t$ and 7-GDD $6^{t+1}$ (Lemma 3.5) to get a 7-GDD $(6m)^t(6r)^1$. Apply Lemma 4.1 for the desired PBD. □

**Lemma 4.14** *Let* $7 \in B(K)$*, suppose there is a* $TD(50, m)$ *and* $0 \leq x, y \leq m$*. If there exist an* $IPBD(6m + f, f, K)$*, and*

*a) an* $IPBD(6x + f, f, K)$ *and a* $PBD(6m + 42y + f, K)$*, or*

*b) an* $IPBD(6m + 42y + f, f, K)$ *and a* $PBD(6x + f, K)$*,*

*then there exists a* $PBD(294m + 6x + 42y + f, K)$*.*

**Proof.** Truncate one group of the TD$(50, m)$ to obtain a $\{49, 50\}$-GDD $m^{49}x^1$. Giving weight 48 to $y$ points of one untruncated group and weight 6 to all other points Wilson's Fundamental Construction yields a 7-GDD $(6m)^{48}(6x)^1(6m + 42y)^1$ where we use as ingredient GDDs a 7-GDD $6^{49}$, a 7-GDD $6^{50}$ (from Lemma 3.5), a 7-GDD $6^{48}48^1$ and a 7-GDD $6^{49}48^1$ (from Corollary 3.9). Applying Lemma 4.1 completes the proof. □

**Lemma 4.15** *Let $7 \in B(K)$, suppose there is a TD$(50, m)$ and $0 \leq x \leq 1$, $0 \leq y \leq 3$, $0 \leq z \leq m - x - y$. If there exist an IPBD$(6m + f, f, K)$ and a PBD$(6x + 12y + 48z + f, f, K)$, then there exists a PBD$(294m + 6x + 12y + 48z + f, K)$.*

**Proof.** Truncate one group to size $x + y + z$ and give weight 48 to $z$, weight 12 to $y$ and weight 6 to $x$ points of the truncated group. All remaining points get weight 6. Now, Wilson's Fundamental Construction yields a 7-GDD $(6m)^{49}(6x + 12y + 48z)^1$ where we use as ingredient GDDs a 7-GDD $6^{49}$, a 7-GDD $6^{50}$ (from Lemma 3.5), a 7-GDD $6^{49}12^1$ (from Lemma 3.6) and a 7-GDD $6^{49}48^1$ (from Corollary 3.9). Again, applying Lemma 4.1 completes the proof. □

**Lemma 4.16** *Let $\{7, 13\} \subset B(K)$ and suppose there is a TD$(n, m)$ with $51 \leq n$. There is a PBD$(v, K)$ with*

*a)* $v = 294m + 6t + 6r + 1$ *if* $0 \leq r \leq t \leq m$ *and* $\{6m + 1, 6t + 1, 6r + 1\} \subset B(K)$, *containing flats of order* $7, 6m + 1, 6t + 1, 6r + 1$ *and if* $r > 0$ *a flat of order* 13*;*

*b)* $v = 294(m-1)+6t+6r+1$ *if* $0 \leq r < m$, $49 \leq t < n$, $\{6m+1, 6r+1\} \subset B(K)$ *and* $6t + 1 \in B(\{7\})$, *containing flats of order* $7, 6m + 1, 6r + 1$ *and if* $r > 0, t > 49$ *a flat of order* 13*;*

*c)* $v = 294(m-1)+6t+6r-5$ *if* $1 \leq r \leq m$, $50 \leq t \leq n$, $\{6m+1, 6r+1\} \subset B(K)$ *and* $6t + 1 \in B(\{7\})$, *containing flats of order* $7, 6m + 1, 6r + 1$ *and if* $r > 1, t > 50$ *a flat of order* 13*;*

*d)* $v = 294(m - 2) + 6t + 6r + 1$ *if* $49 \leq r \leq t \leq n$, $6m + 1 \in B(K)$ *and* $\{6t + 1, 6r + 1\} \subset B(\{7\})$, *containing flats of order* $7, 6m + 1$ *and if* $r > 49$ *a flat of order* 13*;*

*e)* $v = 294(m - 2) + 6t + 6r - 5$ *if* $50 \leq r \leq t \leq n$, $6m + 1 \in B(K)$ *and* $\{6t + 1, 6r + 1\} \subset B(\{7\})$, *containing flats of order* $7, 6m + 1$ *and if* $r > 50$ *a flat of order* 13*;*

*f)* $v = 300(m - 1) + 6t + 6r + 1$ *if* $0 \leq r < 50$, $0 \leq t < m$, $\{6m + 1, 6m - 5, 6t + 1\} \subset B(K)$ *and* $6r + 1 \in B(\{7\})$, *containing flats of order* $7, 6m - 5, 6t + 1$, *and if* $t > 0$ *a flat of order* 13, *and if* $r > 0$ *a flat of order* $6m + 1$*;*

*g)* $v = 300(m - 1) + 6t + 6r - 5$ *if* $1 \leq r < 51$, $1 \leq t \leq m$, $\{6m + 1, 6m - 5, 6t + 1\} \subset B(K)$ *and* $6r + 1 \in B(\{7\})$, *containing flats of order* $7, 13, 6m - 5, 6t + 1$ *and if* $r > 1$ *a flat of order* $6m + 1$*;*

*h)* $v = 300(m-2)+6t+6r+1$ *if* $0 \le r < 50 \le t \le n$, $\{6m+1, 6m-5\} \subset B(K)$ *and* $\{6t+1, 6r+1\} \subset B(\{7\})$, *containing flats of order* $7, 6m-5$, *and if* $r > 0, t > 50$ *a flat of order* $13$, *and if* $r > 0$ *a flat of order* $6m+1$;

*i)* $v = 300(m-2)+6t+6r-5$ *if* $1 \le r < 51 \le t \le n$, $\{6m+1, 6m-5\} \subset B(K)$ *and* $\{6t+1, 6r+1\} \subset B(\{7\})$, *containing flats of order* $7, 13, 6m-5$ *and if* $r > 1$ *a flat of order* $6m+1$.

**Proof.** First note that a TD$(49, m)$, TD$(50, m)$ or TD$(51, m)$ is embedded in the TD$(n, m)$. So, for a) truncate two groups of a TD$(51, m)$ to get a $\{49, 50, 51\}$-GDD $m^{49}t^1r^1$. Give weight 6 to all points in Wilson's Fundamental Construction and use the following ingredient GDDs, a 7-GDD $6^{49}$, a 7-GDD $6^{50}$ from Lemma 3.5 and a $\{7, 13\}$-GDD $6^{51}$ from Lemma 3.6, to obtain a $\{7, 13\}$-GDD $(6m)^{49}(6t)^1(6r)^1$. Now adjoining an infinite point and filling groups gives the desired PBD. Note that the existence of a flat of order 13 is guaranteed only if there is a block of size 51 in the master GDD, i.e. only if $r, t > 0$.

The construction in the other cases is similar (always weight 6 for the points) so that we only need to find the master GDD in each case. Ingredient GDDs 7-GDD $6^t$ and 7-GDD $6^r$ exist by Lemma 3.5 if required.

For b/c) spike(=extend) one line of a TD$(50, m)$ to size $t$, then truncate a group to size $r$. If the $t$ line and $r$ group intersect in a deleted point we get b) a $\{49, 50, 51, t\}$-GDD $m^{49}r^1 1^{t-49}$, otherwise c) a $\{49, 50, 51, t\}$-GDD $m^{49}r^1 1^{t-50}$.

For d/e) we spike two lines of a TD$(49, m)$ to size $t$ or $r$, respectively. Here we can assume the groups on the spiked lines coincide as much as possible. There are slight differences if spiked lines intersect within the TD or on the spikes. Within the TD we get d) a $\{49, 50, 51, r, t\}$-GDD $m^{49}2^{r-49}1^{t-r}$, while on the spikes e) a $\{49, 50, 51, r, t\}$-GDD $m^{49}2^{r-50}1^{t-r+1}$ is obtained.

For types f/g), we truncate a group of a TD$(51, m)$ to size $t$. Then we truncate a block to size $r$ where we do not delete points from the truncated group. If we truncate a 50-block we get f) a $\{49, 50, 51, r\}$-GDD $m^r(m-1)^{50-r}t^1$ (so we need $t < m$ for a group truncation). Truncating a 51-block gives g) a $\{49, 50, 51, r\}$-GDD $m^{r-1}(m-1)^{51-r}t^1$ (so we need $0 < t$).

For types h/i), we spike one line of a TD$(50, m)$ to size $t$. Then we truncate a block to size $r$ of the spiked TD where we do not delete points from the spike. Again, there are slight differences if we truncate a 50-block to obtain g) a $\{49, 50, 51, r, t\}$-GDD $m^r(m-1)^{50-r}1^{t-50}$ or a 51-block to get i) a $\{49, 50, 51, r, t\}$-GDD $m^{r-1}(m-1)^{51-r}1^{t-50}$ (so we need $51 \le t$ for a spike).
□

If the master GDDs above do not have groups of order 7 or 13 we can successfully extend Lemma 4.16 by applying Lemma 4.1 on the master GDDs.

**Lemma 4.17** *Let* $\{7,13\} \subset B(K)$ *and suppose there is a* $TD(n,m)$ *with* $51 \leq n$.

*a) If* $0 \leq r \leq t \leq m$, *there is a* $PBD(6m+f,K)$ *containing a flat of order* $f$ *and there are a* $PBD(6t+f,K)$ *and a* $PBD(6r+f,K)$ *of which at least one PBD contains a flat of order* $f$, *then a* $PBD(294m+6t+6r+f,K)$ *exists.*

*f) If* $0 \leq r < 50$, $0 \leq t < m$, *there are* $PBD(6m+f,K)$ *and* $PBD(6m-6+f,K)$ *containing flats of order* $f$, $6t+f \in B(K)$ *and* $6r+1 \in B(\{7\})$, *then a* $PBD(300(m-1)+6t+6r+f,K)$ *exists.*

*g) If* $1 \leq r < 51$, $1 \leq t \leq m$, *there are* $PBD(6m+f,K)$ *and* $PBD(6m-6+f,K)$ *containing flats of order* $f$, $6t+f \in B(K)$ *and* $6r+1 \in B(\{7\})$, *then a* $PBD(300(m-1)+6t+6r-6+f,K)$ *exists.*

**Lemma 4.18** *Let* $\{7,13\} \subset B(K)$ *and let* $q \geq 53$ *be a prime power. Suppose there are a* $PBD(6q+f,K)$ *and a* $PBD(6(q-1)+f,K)$ *both with a flat of order* $f$. *Then there exists a* $PBD(306q-6x+f,K)$ *for* $0 \leq x \leq 51$. *If in addition* $q \geq 101$ *and there is a* $PBD(6(q-2)+f,K)$ *with a flat of order* $f$, *then there exists a* $PBD(306q-6x-12y+f,K)$ *for* $0 \leq x+y \leq 51$.

**Proof.** Consider an oval (or hyperoval if $q$ is even) in a projective plane $\mathrm{PG}(2,q)$ and delete a point from the (hyper)oval. A $\mathrm{TD}(q+1,q)$ is obtained in which each group contains at most one point from the (hyper)oval. Deleting groups yields a $\mathrm{TD}(51,q)$ where all groups have exactly one (hyper)oval point. Delete $x$ (hyper)oval points to get a $\{49,50,51\}$-GDD $q^{51-x}(q-1)^x$ which in turn with Wilson's Fundamental Construction (weight 6 to all points, 7-GDD $6^{49}$, 7-GDD $6^{50}$, $\{7,13\}$-GDD $6^{51}$) provides a $\{7,13\}$-GDD $(6q)^{51-x}(6(q-1))^x$. Finally, apply Lemma 4.1 to obtain the desired $\mathrm{PBD}(306q-6x+f,K)$.

Now delete a non-tangent point (or just a non-hyperoval point if $q$ is even) from $\mathrm{PG}(2,q)$ to obtain a $\mathrm{TD}(q+1,q)$ in which there are exactly $(q+1)/2$ groups containing no oval point and $(q+1)/2$ groups containing exactly two oval points (or exactly $q/2$ groups containing no hyperoval point and $q/2+1$ groups containing exactly two hyperoval points if $q$ is even). Since we require $q \geq 101$ we are able to delete groups in such a way that we get a $\mathrm{TD}(51,q)$ where all groups have exactly two (hyper)oval points. Now delete from $x$ groups exactly one (hyper)oval point and from $y$ groups two (hyper)oval points for a $\{49,50,51\}$-GDD $q^{51-x-y}(q-1)^x(q-2)^y$. Again, via Wilson's Fundamental Construction and Lemma 4.1 the desired $\mathrm{PBD}(306q-6x-12y+f,K)$ is obtained. □

We also need the *singular indirect product* construction which we take from [23].

| | $u$ | $v$ | $w$ | PBD with flat | $a$ |
|---|---|---|---|---|---|
| 1063 | 7 | 169 | 25 | TD(7,24)+$\infty$ | 20 |
| 1069 | 7 | 169 | 25 | TD(7,24)+$\infty$ | 19 |
| 1567 | 7 | 253 | 37 | TD(7,36)+$\infty$ | 34 |
| 1579 | 7 | 253 | 37 | TD(7,36)+$\infty$ | 32 |

Table 2: Proof of Corollary 4.20

**Lemma 4.19** *If there is an IPBD$(v, w, K)$, $u \in K$, $0 \leq a \leq w \leq v$, the incomplete transversal design ITD$(u; v-a, w-a)$ exists, and $u(w-a)+a \in B(K)$, then there is a PBD$(u(v-a)+a, K)$ containing flats of order $u$ and $u(w-a)+a$.*

As a first application of the singular indirect product we obtain the following.

**Corollary 4.20** $1063, 1069, 1567$ *and* $1579$ *are not essential in a Wilson basis for $N_{1,6}$.*

**Proof.** Use the previous lemma to show that $\{1063, 1069, 1567, 1579\} \subset B(\{7, 25, 37, 55, 61, 67\})$. The details are given in Table 2. The needed ITDs are all listed in [5]; they can also be constructed by Lemma 2.5 with $m = 16$ or 8. $\square$

In order to apply the constructions above we need as many ingredient designs as possible. So we present in addition to the designs from Section 3 some new (resolvable) BIBDs for $k = 6, 12, 13$ by providing suitable base blocks.

**Lemma 4.21** *If $834 \leq t \leq 5460$, $t$ is even and $q = 6t+1$ is a prime power, then an RBIBD$(30t+6, 6, 1)$ exists and hence an IPBD$(36t+7, 6t+1, \{7\})$.*

**Proof.** As in [16] for $t \leq 832$ we construct $2t$ base blocks in $GF(5) \times GF(q)$. Let $\omega$ be a generator of the multiplicative group $GF(q)^*$ and $d = (q-1)/2$. We define blocks

$$S_0 = \{(0, \omega^0), (0, \omega^d), (1, \omega^{\gamma_1}), (1, \omega^{\gamma_1+d}), (4, \omega^{\gamma_2}), (4, \omega^{\gamma_2+d})\}$$

and

$$S_1 = \{(0, \omega^3), (0, \omega^{3+d}), (2, \omega^{\gamma_1+3}), (2, \omega^{\gamma_1+3+d}), (3, \omega^{\gamma_2+3}), (3, \omega^{\gamma_2+3+d})\}.$$

The base blocks are $S_{b,a} = (1, \omega^{6a}) \cdot S_b$ with $b = 0, 1$ and $a = 0, 1, \ldots, t-1$. It remains to specify $\gamma_1, \gamma_2$ such that the pure and mixed differences are

evenly spread amongst the 6 cyclotomic classes. This is done in the appendix Table 8. Now with a new point $\infty$ adjoined and a new base block $\{\infty, (i,0) : i = 0, 1, \ldots, 4\}$ an RBIBD$(30t + 6, 6, 1)$ is obtained where a resolution set is formed by the partial development of the base blocks $S_{b,a} \mod (5, -)$ augmented with the new base block. The development of this resolution set $\mod(-, q)$ generates the other resolution sets. $\square$

The necessary conditions for the existence of a BIBD$(v, 13, 1)$ are $v \equiv 1$ or 13 mod 156. Until now, only a few BIBD$(v, 13, 1)$ with $v$ reasonably small were known to exist. The following BIBDs of order $q$, $q \equiv 1$ mod 156 a prime power, are constructed by difference families using an approach described in [15].

**Lemma 4.22** *BIBD$(q, 13, 1)$ exist for* $q \in A = \{6241,\ 8737,\ 9829,\ 14197,\ 15601,\ 16069,\ 16381,\ 16693,\ 18097,\ 19813,\ 20593,\ 20749,\ 21061,\ 21529,\ 21841,\ 21997,\ 22153,\ 22621,\ 22777,\ 23557,\ 23869,\ 24181,\ 24337,\ 25117,\ 25741,\ 26053,\ 26209,\ 26833,\ 27457,\ 28081,\ 28549,\ 29017,\ 29173,\ 29641,\ 30109,\ 30577,\ 31357,\ 31513,\ 31981\}$.

**Proof.** We construct base blocks in $GF(q)$. Let $w$ be a cube root of unity and let $\{m, c, c', c''\} \subset GF(q)$. We define

$$S = \{0, 1, w, w^2, c, cw, cw^2, c', c'w, c'w^2, c'', c''w, c''w^2\}$$

and $S_i = m^i S$ with $i = 0, 1, \ldots, (q-1)/156$. With the values for $m, w, c, c', c''$ provided in the appendix Table 9 it is easy to check that every distance is covered exactly once by the blocks $S_i$. Note that if $q = p^n$ is a proper prime power, then an element $x \in GF(q), x = \sum_{i=0}^{n-1} a_i \omega^i$ is represented as $x = \sum_{i=0}^{n-1} a_i p^i$ where $\omega$ is a root of the primitive polynomial for $q$ from Table 7. $\square$

We can make the above difference families block disjoint by using Abel's adder.

**Lemma 4.23 (Abel [1])** *Suppose $q = k(k-1)t + 1$ is a prime power and a $(q, k, 1)$ difference family over GF(q) is given by $B_i = m^i B_0$ for $i = 0, 1, \ldots, t-1$. Then there is a block disjoint $(q, k, 1)$ difference family over GF(q).*

**Proof.** Define a new difference family by $C_i = B_i + cm^i$. We will now show there is a choice of $c$ such that $C_i$ for $i = 0, 1, \ldots, t-1$ is the block disjoint difference family. Let $B_0 = (b_1, b_2, \ldots, b_k)$. Now $C_i = m^i(B_0 + c)$. Clearly, we cannot have $0 \in (B_0 + c)$, so this prohibits $k$ values of $c$. If $C_i$ and $C_j$ have a common element and $i > j$, then $m^i b_y + cm^i = m^i b_z + cm^j$ for some $y \neq z$, i.e., $c = (b_z - m^{i-j} b_y)(m^{i-j} - 1)^{-1}$. This eliminates at most a further

$k(k-1)$ values of $c$ for each value of $i-j$. So we have prohibited at most $q-(k-1)^2$ values of $c$, and there are at least $(k-1)^2$ values that work. □

So we may again apply the Ray-Chaudhuri-Wilson construction to get RBIBDs.

**Lemma 4.24 ([24])** *Let the set $A$ be defined as in Lemma 4.22. If $q \in A$, then there is an RBIBD$(13q, 13, 1)$.*

The following family of 13-GDDs is particularly important as it gives the smallest PBDs in each fibre $19, 25, 31$ or $37 \bmod 42$.

**Lemma 4.25** *If $t = 1, 10, 13, 14, 19, 23, 30, 33, 44, 50, 51, 55, 59, 61, 63$ or $69 \le t \le 2730$, and $q = 12t+1$ is a prime power, then a 13-GDD $13^q$ exists and hence a PBD$(156t+13, \{13\})$.*

**Proof.** For $t = 1$ just take a TD$(13, 13)$. For $t = 14$ replace the groups of a TD$(13, 169)$ by the 13-GDD $13^{13}$ just obtained. For $t \neq 1, 14$ we construct $t$ base blocks in $GF(13) \times GF(q)$. Let $\omega$ be a generator of the multiplicative group $GF(q)^*$. We define a block of size 13

$$S = \{(0,0)\} \cup \{(1, \pm\omega^0), (3, \pm\omega^{\gamma_1}), (4, \pm\omega^{\gamma_2}), (9, \pm\omega^{\gamma_3}), (10, \pm\omega^{\gamma_4}), (12, \pm\omega^{\gamma_5})\}.$$

The base blocks are $S_a = (1, \omega^{6a}) \cdot S$ with $a = 0, 1, \ldots, t-1$. With $\gamma_1, \ldots, \gamma_5$ from the appendix Table 10 we get a 13-GDD $13^q$ where the groups are $\{(i, x) : i = 0, 1, \ldots, 12\}$ for $x \in GF(q)$. If we consider the groups to be blocks of size 13 we obtain the desired PBD. □

Moreover, we constructed block disjoint difference families of order $q$, $q \equiv 1 \bmod 132$ a prime power, with block size 12 which provide resolvable BIBDs of size 12 when we apply Ray-Chaudhuri-Wilson construction.

**Lemma 4.26** *BIBD$(q, 12, 1)$ exist for $q \in B = \{5413, 6073, 6337, 6469, 6733, 6997, 7129, 7393, 7789, 7921, 8053, 8317, 8581, 8713, 9109, 9241, 9769, 9901, 10429, 10957, 11353, 11617, 11881, 12277, 12409, 12541, 13597, 13729, 14389, 14653, 15313, 15973, 16369, 16633, 17029, 17161, 17293, 18217, 18481, 19009, 19141, 19273, 19801, 20593, 20857, 21121, 21517, 21649, 22441, 22573, 23497, 23629, 23761, 23893, 24421, 25609, 25741, 25873, 27061, 27457, 28513, 28909, 29173, 29437, 29569, 29833, 30097, 30493, 30757, 31153, 32077, 32341\}$. If $q \in B$, then there is a BIBD$(q, 12, 1)$, an RBIBD$(12q, 12, 1)$ and an IPBD$(12q + (12q-1)/11, (12q-1)/11, \{13\})$.*

**Proof.** We construct base blocks in $GF(q)$. Let $w$ be a cube root of unity and let $\{m, c, c', c''\} \subset GF(q)$. We define

$$S = \{1, w, w^2, c, cw, cw^2, c', c'w, c'w^2, c'', c''w, c''w^2\}$$

and $S_i = m^i S$ with $i = 0, 1, \ldots, (q-1)/132$. With the values for $m, w, c, c', c''$ provided in the appendix Tables 11 and 12 it is easy to check that every distance is covered exactly once by the blocks $S_i$. $\square$

**Lemma 4.27** *If $t \notin \{30, 33\}$, $29 \leq t \leq 2730$, and $q = 12t + 1$ is a prime power, then an RBIBD$(132t + 12, 12, 1)$ exists and hence an IPBD$(144t + 13, 12t + 1, \{13\})$.*

**Proof.** Here, we construct $t$ base blocks in $GF(11) \times GF(q)$. Let $\omega$ be a generator of the multiplicative group $GF(q)^*$. We define a block of size 12

$$S = \{(0, \pm\omega^0), (1, \pm\omega^{\gamma_1}), (3, \pm\omega^{\gamma_2}), (4, \pm\omega^{\gamma_3}), (5, \pm\omega^{\gamma_4}), (9, \pm\omega^{\gamma_5})\}.$$

The base blocks are $S_a = (1, \omega^{6a}) \cdot S$ with $a = 0, 1, \ldots, t-1$. With $\gamma_1, \ldots, \gamma_5$ from the appendix Table 13 and the additional base block $\{\infty, (i, 0) : i = 0, 1, \ldots, 10\}$ an RBIBD$(132t + 12, 12, 1)$ is obtained. $\square$

We actually tried all prime powers $< 32768$ in the appropriate residue class for our direct constructions. In Lemma 4.22, we had failures for $q = 156t + 1$ with $q \in \{157, 313, 625, 937, 1093, 1249, 1873, 2029, 2341, 2809, 3121, 3433, 4057, 4993, 6397, 6553, 6709, 7177, 7333, 7489, 8269, 8581, 8893, 9049, 10141, 10453, 10609, 11701, 12637, 13417, 13729, 14821, 15289, 15913, 17161, 17317, 18253, 19501, 24649, 28393, 32761\}$. Thus we failed in 41 of the 80 possible cases. Greig [15] also looked at cosets of size 13, with only success for $q = 21061$ (which adds nothing new here). We will get a quick failure if $\log(x^{52t} - 1) \equiv 0 \bmod 26$, and we had 5 quick failures. In Lemma 4.26, we had failures for $q = 132t + 1$ with $q \in \{397, 529, 661, 1321, 1453, 1849, 2113, 2377, 3037, 3169, 3301, 3433, 3697, 4093, 4357, 4489, 4621, 5281\}$. Thus we failed in 18 of the 90 possible cases. In Lemma 4.27, we had failures for $q = 12t + 1$ with $t \in \{1$–$6, 8$–$10, 13$–$16, 19, 20, 23, 24, 26, 28, 30, 33\}$. In Lemma 4.25, we had failures for $q = 12t + 1$ with $t \in \{2$–$6, 8, 9, 15, 16, 20, 24, 26, 28, 31, 34$–$36, 38, 45, 48, 52, 56, 64\}$.

# 5 PBD-Closure of $K = \{7, 13\}$

In this section, we show that all positive integers $v \equiv 1 \bmod 6$ are in $B(\{7, 13\})$ with the possible exceptions in $Q_{\{7,13\}}$. $Q_{\{7,13\}}$ contains 3960 elements which are listed in the appendix (Table 14). To reduce the problem to a finite one we first construct a representative PBD$(v, \{7, 13\})$ in each possible residue class $r$ modulo $(14 \cdot 84)$, $r \not\equiv 1, 7 \bmod 42$.

**Lemma 5.1** *Let $R = \{r_{i,j} : i = 0, 1, \ldots, 13, j = 13, 19, \ldots, 79\}$ be given by the entries in Table 3. If $r \in R$, then $r \in B(\{7, 13\})$.*

| 13 | 19 | 25 | 31 | 37 | 55 | 61 | 67 | 73 | 79 |
|---|---|---|---|---|---|---|---|---|---|
| 13 | 12955 | 12961 | 21199 | 21205 | 3583 | 14173 | 24763 | 3601 | 3607 |
| 2449 | 13039 | 27157 | 21283 | 9529 | 4843 | 18961 | 18967 | 7213 | 18979 |
| 3709 | 15475 | 10777 | 21367 | 19021 | 3751 | 16693 | 17875 | 14353 | 13183 |
| 2617 | 16735 | 16741 | 19099 | 4993 | 307 | 14425 | 16783 | 15613 | 19147 |
| 3877 | 14467 | 6241 | 21535 | 19189 | 8623 | 1573 | 18043 | 14521 | 12175 |
| 8665 | 13375 | 21613 | 19267 | 5161 | 8707 | 15769 | 19303 | 13429 | 19315 |
| 517 | 14635 | 18169 | 19351 | 17005 | 559 | 14677 | 17035 | 19393 | 19399 |
| 601 | 15895 | 19429 | 19435 | 2977 | 4171 | 12409 | 15943 | 19477 | 19483 |
| 5389 | 12451 | 19513 | 19519 | 17173 | 8959 | 12493 | 17203 | 7801 | 3103 |
| 4297 | 12535 | 16069 | 29011 | 13729 | 4339 | 11401 | 24343 | 21997 | 23179 |
| 2029 | 13795 | 2041 | 29095 | 14989 | 2071 | 17365 | 16195 | 7969 | 12679 |
| 2113 | 20935 | 16237 | 28003 | 13897 | 5683 | 9217 | 20983 | 6877 | 20995 |
| 2197 | 13963 | 15145 | 24559 | 21037 | 6943 | 8125 | 16363 | 21073 | 17551 |
| 1105 | 15223 | 16405 | 24643 | 9361 | 1147 | 21145 | 16447 | 4693 | 21163 |

Table 3: Representative PBD$(r_{i,j}, \{7, 13\})$ with $r_{i,j} \equiv 84i + j \bmod 1176$

**Proof.** We give for each entry from the table a construction and the parameters used. All requisite (I)PBDs are easily obtained from Lemmas 3.1, 3.3, 4.2 or are constructed within this proof. Orders which do not occur in the table but are required as components are marked with an asterisk.

An IPBD of order $3895^*$ with a hole of size 649 exists by Lemma 3.2 with $t = 18$. The entries 3103, 3607, 12175, 12679, 13183 are from Lemma 3.3 with $t = 86$, 100, 338, 352, 366. Orders 4993, 7801, 9361, 13729, 14353, 17005, 17551, 19189, 19267 are constructed by Lemma 4.2 with $k = 13$, $n = 384$, 600, 720, 1056, 1104, 1308, 1350, 1476, 1482. A PBD of order $3703^*$ with a hole of size 517 exists by Corollary 4.5 with $m = 43$, $t = 15$. Moreover, $2401^*$, 2449 are obtained from Lemma 4.10 with $n = 7$, $m = 50, 51$ and 8707 comes from Lemma 4.11 with $m = 97$, $t = 91$, $f = 13$. 4339, 8665 are from Lemma 4.12 with $m = 49$, $f = 7$ and $m = 97$, $f = 13$. 16447, 28003 are from Lemma 4.15 with $m = 49$, $x = 0$, $y = 2$, $z = 42$, $f = 1$ and $m = 83$, $x = 0$, $y = 3$, $z = 74$, $f = 13$. From Lemma 4.17.a we get 24559, 24643 with $m = 83$, $r = 12$, $t = 12, 26$, $f = 13$ and 29011, 29095 with $m = 97$, $r = 12, 26$, $t = 68$, $f = 13$. Furthermore, 19303, 19315, 19351, 19393, 19399, 19429, 19435, 19477, 19483, 19513, 19519, 21535, 21613 are from Lemma 4.18 with $q = 64$, $f = 1$, $47 \geq x \geq 11$ or $q = 71$, $f = 1$, $x = 32, 19$. The entries 6241, 16069, 16693, 21997 are taken from Lemma 4.22. Moreover, 1573, 2041, 2197, 2977, 3601, 4693, 5161, 6877, 7969, 9217, 9529, 10777, 11401, 12493, 12961, 13429, 13897, 14521, 14677, 14989, 15613, 15769, 16237, 17173, 21073,

27157 are from Lemma 4.25 with $t = 10$, 13, 14, 19, 23, 30, 33, 44, 51, 59, 61, 69, 73, 80, 83, 86, 89, 93, 94, 96, 100, 101, 104, 110, 135, 174. 7213 is from Lemma 4.27 with $t = 50$. Finally, all remaining orders $r$ can be constructed using Lemma 4.19 (Singular Indirect Product) or Lemma 4.16. The parameters of these constructions are presented in Table 4 and Table 5. The needed ITDs can all be constructed by Lemmas 2.5–2.9. □

Table 4: Applications of the Singular Indirect Product

| $r$ | $u$ | $v$ | $w$ | $a$ | $r$ | $u$ | $v$ | $w$ | $a$ |
|---|---|---|---|---|---|---|---|---|---|
| 307 | 7 | 49 | 7 | 6 | 517 | 7 | 85 | 13 | 13 |
| 559 | 7 | 85 | 7 | 6 | 601 | 7 | 91 | 7 | 6 |
| 1105 | 7 | 169 | 13 | 13 | 1147 | 7 | 169 | 7 | 6 |
| 1387* | 7 | 223 | 37 | 29 | 1471* | 7 | 223 | 37 | 15 |
| 1483* | 7 | 217 | 7 | 6 | 2029 | 7 | 295 | 7 | 6 |
| 2071 | 7 | 301 | 7 | 6 | 2113 | 7 | 307 | 7 | 6 |
| 2437* | 7 | 367 | 61 | 22 | 2617 | 7 | 379 | 7 | 6 |
| 3583 | 7 | 517 | 7 | 6 | 3709 | 7 | 583 | 97 | 62 |
| 3751 | 13 | 295 | 7 | 7 | 3877 | 7 | 559 | 7 | 6 |
| 4171 | 7 | 601 | 7 | 6 | 4297 | 7 | 631 | 91 | 20 |
| 4843 | 7 | 727 | 121 | 41 | 5389 | 7 | 871 | 145 | 118 |
| 5683 | 7 | 871 | 145 | 69 | 6943 | 7 | 1087 | 181 | 111 |
| 8125 | 13 | 637 | 13 | 13 | 8623 | 7 | 1375 | 229 | 167 |
| 8959 | 7 | 1447 | 241 | 195 | 12409 | 7 | 1807 | 259 | 40 |
| 12451 | 7 | 1813 | 259 | 40 | 12535 | 7 | 1879 | 313 | 103 |
| 12955 | 7 | 1951 | 325 | 117 | 13039 | 13 | 1015 | 13 | 13 |
| 13375 | 7 | 2023 | 337 | 131 | 13795 | 7 | 2095 | 349 | 145 |
| 13963 | 7 | 2065 | 295 | 82 | 14173 | 7 | 2101 | 301 | 89 |
| 14425 | 7 | 2143 | 307 | 96 | 14467 | 7 | 2149 | 307 | 96 |
| 14635 | 7 | 2239 | 373 | 173 | 15145 | 13 | 1177 | 13 | 13 |
| 15223 | 13 | 1183 | 13 | 13 | 15475 | 7 | 2383 | 397 | 201 |
| 15895 | 7 | 2455 | 409 | 215 | 15943 | 7 | 2383 | 397 | 123 |
| 16195 | 7 | 2359 | 337 | 53 | 16363 | 7 | 2455 | 409 | 137 |
| 16405 | 7 | 2395 | 343 | 60 | 16735 | 7 | 2599 | 433 | 243 |
| 16741 | 7 | 2401 | 301 | 11 | 16783 | 7 | 2527 | 421 | 151 |
| 17035 | 7 | 2437 | 295 | 4 | 17203 | 7 | 2599 | 433 | 165 |
| 17365 | 13 | 1513 | 217 | 192 | 17875 | 13 | 1387 | 13 | 13 |
| 18043 | 7 | 2743 | 457 | 193 | 18169 | 7 | 2689 | 385 | 109 |
| 18967 | 13 | 1471 | 13 | 13 | 20983 | 7 | 3247 | 541 | 291 |
| 23179 | 7 | 3463 | 577 | 177 | 24343 | 7 | 3703 | 517 | 263 |
| 24763 | 7 | 3895 | 649 | 417 | | | | | |

Table 5: Applications of Lemma 4.16

| $r$ | Case | $n$ | $m$ | $t$ | $r$ | $r$ | Case | $n$ | $m$ | $t$ | $r$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 18961 | f) | 65 | 64 | 2 | 8 | 18979 | i) | 65 | 64 | 56 | 8 |
| 19021 | c) | 65 | 64 | 56 | 28 | 19099 | i) | 65 | 64 | 56 | 28 |
| 19147 | g) | 65 | 64 | 14 | 28 | 20935 | a) | 51 | 71 | 2 | 8 |
| 20995 | c) | 72 | 71 | 56 | 14 | 21037 | c) | 72 | 71 | 63 | 14 |
| 21145 | a) | 51 | 71 | 2 | 43 | 21163 | i) | 72 | 71 | 63 | 15 |
| 21199 | i) | 72 | 71 | 56 | 28 | 21205 | c) | 72 | 71 | 56 | 49 |
| 21283 | i) | 72 | 71 | 70 | 28 | 21367 | i) | 72 | 71 | 63 | 49 |

**Lemma 5.2** *Let $m \equiv 1$ mod 14, $3501 \leq m \leq 4901$. If $v = 12m + 1$, then $v \in B(\{7, 13\})$.*

**Proof.** This follows by application of Lemma 4.14 with parameters $m = 139$, $x = 50$, $y = 20, 24, \ldots, 136$, $f = 7$; $m = 139$, $x = 99$, $y = 133, 137$, $f = 7$; $m = 167$, $x = 50$, $y = 0, 4, \ldots, 164$, $f = 7$ and $m = 181$, $x = 50$, $y = 2, 6, \ldots, 126$, $f = 7$ which gives $v$ in the intervals $42013 - 46885$, $47053-47221$, $49405-56293$ and $53605-58813$. The existence of all requisite PBDs is ensured by Lemma 3.1. The remaining values from the interval $47389 - 49237$ are obtained from Corollary 4.5 with parameters $m = 539$, $t = 352, 408, 436, 492$; $m = 540$, $t = 534$; $m = 560$, $t = 86, 338, 366$; $m = 574$, $t = 86$; Corollary 4.7 with $u = 85$, $m = 547$, $t = 317, 401$ and Lemma 4.25 with $t = 308$. Here, the requisite PBDs exist by Lemma 3.1, Lemma 5.1, Lemma 4.2 ($k = 7$, $n = 463$), or Lemma 4.19 ($u = 7$, $v = 421$, $w = 7$, $a = 0$; $u = 7$, $v = 463$, $w = 7$, $a = 6$; $u = 7$, $v = 931$, $w = 7$, $a = 6$). □

**Lemma 5.3** *If $v \equiv 1$ mod 6, $v \geq 319825$ then $v \in B(\{7, 13\})$.*

**Proof.** Note $v \equiv 1, 7 \bmod 42$ is covered by Lemma 3.1. For $v \equiv 13, 19, 25, 31, 37 \bmod 42$, there is exactly one $r_{i,j}$ from Table 3 such that $v \equiv 7 \cdot 84 + r_{i,j} \bmod 1176$. If $v \geq 299628 + r_{i,j}$, then there is a unique representation $v = 84m + r_{i,j}$ such that $m \equiv 7 \bmod 14$, $m \geq 3567$. So a TD$(14, m)$ exists by Lemma 2.3 and both a BIBD$(6m+1, 7, 1)$ and a BIBD$(12m+1, 7, 1)$ exist by Lemma 3.1. Thus, by applying Corollary 4.5 with $r_{i,j} = 6t+1$ we can construct every value $v$ exceeding 319819 except those with $v \equiv r \bmod 1176$ where $r \in E = \{199, 235, 241, 247, 283, 367, 451, 535, 655, 697, 955, 1033\}$. The exceptions occur if $t > m$ that is $r_{i,j} > 21403$. In order to apply Corollary 4.5 we need in these cases that $v \geq 15(r_{i,j} - 1) + 1$. Noting that $\max\{r_{i,j}\} = 29095$ we can construct in the exceptional cases every value exceeding 435235.

So it remains to consider the interval $319819 < v \leq 435235$ where $v \equiv r \bmod 1176$, with $r \in E$. Write $v = 84m + r_{i,j}$ such that $m \equiv 1 \bmod 14$, $m \geq 3567$. Again a TD$(14, m)$ exists by Lemma 2.3, a BIBD$(6m + 1, 7, 1)$ exists by Lemma 3.1 and a PBD$(12m + 1, \{7, 13\})$ exists by Lemma 5.2. Hence, if $r_{i,j} \leq 21403$ we can apply Corollary 4.5 to get a PBD$(v, \{7, 13\})$. This works except for $v \equiv 451,955 \bmod 1176$.

Now, let $v \equiv 451,955 \bmod 1176$. There is a representation $v = 84m + r_{4,31} = 84m + 21535$ such that $m \equiv 1, 7 \bmod 14$. If $v \geq 323011$, then we can construct a PBD$(v, \{7, 13\})$ using Corollary 4.5. We are done if we can give a construction for $v = 320323, 320827, 321499, 322003, 322675$: apply Corollary 4.5 with $(m, t) = (3581, 3253)$, $(3588, 3239)$, $(3574, 3547)$, $(3581, 3533)$, $(3588, 3547)$. The requisite PBDs of order $6m + 1$ are obtained from Lemma 4.18 with $q = 71$, $x = 33, 40, 47$, $f = 1$, those of order $12m + 1$ are obtained from Corollary 4.5 with $m = 483$, $t = 386, 400, 414$ and the one of order $6t + 1$ are constructed in Lemma 5.1. $\square$

The next result is obtained with a computer run in which we used all constructions and previously known designs mentioned above to eliminate possible exceptions $v \leq 319819$. There is not enough space to write down all 49369 constructions here, but we provide a web-page

`ftp://ftp.math.uni-rostock.de/pub/members/mgruttm/pbdclosure7_13/index.html`

where for each $v$ at least one construction is given. The computer search left over a set $Q_{\{7,13\}}$ of 3960 possible exceptions listed in Table 14. The largest possible exception is 98683.

**Lemma 5.4** *If $v \equiv 1$ mod 6, $v \leq 319819$, $v \notin Q_{\{7,13\}}$, then $v \in B(\{7, 13\})$.*

Now, our main result follows from Lemma 5.3 and Lemma 5.4.

**Theorem 5.5** *If $v \equiv 1$ mod 6, $v \notin Q_{\{7,13\}}$, then $v \in B(\{7, 13\})$.*

# 6 PBD-Closure of sets $K$ where $\{7, 13\} \subset K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$

Using the same methods as in the previous section we determined the PBD-closure of all sets $K$ where $\{7, 13\} \subset K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$ leaving in each case a number of possible exceptions. The largest exceptions for each $K$ and each fibre $6t + 1$ modulo 42 are represented in Table 6. Again, there is not enough space for the constructions and the sets of exceptions. Hence, we give details at

`ftp://ftp.math.uni-rostock.de/pub/members/mgruttm/pbdclosure7_13/index.html`.

Table 6: Largest possible exception in each residue class $6t+1$ modulo 42 for all sets $K$ with $\{7,13\} \subseteq K \subseteq \{7,13,19,25,31,37,43\}$

| $6t+1$ modulo 42 | 1 | 7 | 13 | 19 | 25 | 31 | 37 |
|---|---|---|---|---|---|---|---|
| $K$ | | | | | | | |
| $\{7,13\}$ | 2605 | 1645 | 14293 | 82549 | 98683 | 91507 | 88447 |
| $\{7,13,19\}$ | 2605 | 805 | 13915 | 13081 | 26191 | 90751 | 18811 |
| $\{7,13,25\}$ | 2605 | 1645 | 12277 | 77635 | 12247 | 22417 | 33133 |
| $\{7,13,19,25\}$ | 2605 | 805 | 12193 | 11065 | 12247 | 18763 | 18685 |
| $\{7,13,31\}$ | 2605 | 1645 | 12277 | 18541 | 24721 | 11833 | 48631 |
| $\{7,13,19,31\}$ | 2605 | 805 | 12193 | 11065 | 18463 | 11077 | 18055 |
| $\{7,13,25,31\}$ | 2605 | 1645 | 12277 | 18541 | 11071 | 11833 | 23305 |
| $\{7,13,19,25,31\}$ | 2605 | 805 | 12193 | 11065 | 11071 | 9859 | 17635 |
| $\{7,13,37\}$ | 1975 | 1435 | 13075 | 48739 | 61807 | 64627 | 13099 |
| $\{7,13,19,37\}$ | 1975 | 805 | 13075 | 10099 | 18757 | 36991 | 13099 |
| $\{7,13,25,37\}$ | 1975 | 1435 | 9799 | 23917 | 9811 | 18217 | 10999 |
| $\{7,13,19,25,37\}$ | 1975 | 805 | 8413 | 9511 | 9811 | 17923 | 6631 |
| $\{7,13,31,37\}$ | 1975 | 1435 | 9799 | 18541 | 24385 | 10111 | 10117 |
| $\{7,13,19,31,37\}$ | 1975 | 805 | 9799 | 10099 | 18379 | 10111 | 10117 |
| $\{7,13,25,31,37\}$ | 1975 | 1435 | 9799 | 17995 | 7417 | 9817 | 7765 |
| $\{7,13,19,25,31,37\}$ | 1975 | 805 | 5935 | 6613 | 6619 | 5995 | 6631 |
| $\{7,13,43\}$ | 2605 | 1645 | 14293 | 82549 | 95785 | 89575 | 82567 |
| $\{7,13,19,43\}$ | 2605 | 805 | 13915 | 13081 | 26191 | 79159 | 18685 |
| $\{7,13,25,43\}$ | 2605 | 1645 | 12277 | 77635 | 12247 | 22417 | 25447 |
| $\{7,13,19,25,43\}$ | 2605 | 805 | 12193 | 11065 | 12247 | 18763 | 18685 |
| $\{7,13,31,43\}$ | 2605 | 1645 | 12277 | 18541 | 24721 | 11833 | 48631 |
| $\{7,13,19,31,43\}$ | 2605 | 805 | 12193 | 11065 | 18463 | 11077 | 17803 |
| $\{7,13,25,31,43\}$ | 2605 | 1645 | 12277 | 17281 | 11071 | 11833 | 23305 |
| $\{7,13,19,25,31,43\}$ | 2605 | 805 | 12193 | 11065 | 11071 | 9859 | 17635 |
| $\{7,13,37,43\}$ | 799 | 1435 | 13075 | 48739 | 60379 | 64627 | 13099 |
| $\{7,13,19,37,43\}$ | 799 | 805 | 13075 | 10099 | 18757 | 36991 | 13099 |
| $\{7,13,25,37,43\}$ | 799 | 1435 | 9799 | 23161 | 9811 | 18217 | 10999 |
| $\{7,13,19,25,37,43\}$ | 799 | 805 | 8413 | 9511 | 9811 | 17923 | 6631 |
| $\{7,13,31,37,43\}$ | 799 | 1435 | 9799 | 18541 | 24385 | 10111 | 10117 |
| $\{7,13,19,31,37,43\}$ | 799 | 805 | 9799 | 10099 | 18379 | 9523 | 10117 |
| $\{7,13,25,31,37,43\}$ | 799 | 1435 | 9799 | 17281 | 7417 | 9817 | 7765 |
| $\{7,13,19,25,31,37,43\}$ | 799 | 805 | 5935 | 6613 | 6619 | 5995 | 6631 |

# 7 Acknowledgements

We wish to thank Charlie Colbourn for providing us the MOLS table in a computer readable format and Julian Abel for many useful comments.

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# A Appendix

The first table gives the primitive polynomials for all proper prime powers 1 mod 12 which are $\leq 2^{15}$. The next six tables give the parameters required in the proof of Lemmas 4.21, 4.22, 4.25. 4.26 (two tables) and 4.27. The entry * indicates that the generating element $\omega$ is a root of the primitive polynomial in the first table. Finally in the last table, we list all integers $v$ for which the existence of a PBD$(v, \{7, 13\})$ is not known.

Table 7: Table of primitive polynomials of $GF(p^n)$ with $p^n \equiv 1 \bmod 12$, $f(x) = b_0x^0 + b_1x^1 + \ldots + b_{n-1}x^{n-1} + x^n$

| $p^n$ | $p$ | $n$ | $b_0, \ldots, b_{n-1}$ | $p^n$ | $p$ | $n$ | $b_0, \ldots, b_{n-1}$ | $p^n$ | $p$ | $n$ | $b_0, \ldots, b_{n-1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 5 | 2 | 2,1 | 625 | 5 | 4 | 3,1,0,1 | 15625 | 5 | 6 | 2,0,0,0,0,1 |
| 49 | 7 | 2 | 3,1 | 2401 | 7 | 4 | 3,0,1,1 | 121 | 11 | 2 | 7,1 |
| 14641 | 11 | 4 | 8,0,0,1 | 169 | 13 | 2 | 2,1 | 2197 | 13 | 3 | 2,0,1 |
| 28561 | 13 | 4 | 6,0,1,1 | 289 | 17 | 2 | 3,1 | 361 | 19 | 2 | 2,1 |
| 529 | 23 | 2 | 7,1 | 841 | 29 | 2 | 3,1 | 961 | 31 | 2 | 12,1 |
| 1369 | 37 | 2 | 5,1 | 1681 | 41 | 2 | 12,1 | 1849 | 43 | 2 | 3,1 |
| 2209 | 47 | 2 | 13,1 | 2809 | 53 | 2 | 5,1 | 3481 | 59 | 2 | 2,1 |
| 3721 | 61 | 2 | 2,1 | 4489 | 67 | 2 | 12,1 | 5041 | 71 | 2 | 11,1 |
| 5329 | 73 | 2 | 11,1 | 6241 | 79 | 2 | 3,1 | 6889 | 83 | 2 | 2,1 |
| 7921 | 89 | 2 | 6,1 | 9409 | 97 | 2 | 5,1 | 10201 | 101 | 2 | 3,1 |
| 10609 | 103 | 2 | 5,1 | 11449 | 107 | 2 | 5,1 | 11881 | 109 | 2 | 6,1 |
| 12769 | 113 | 2 | 10,1 | 16129 | 127 | 2 | 3,1 | 17161 | 131 | 2 | 14,1 |
| 18769 | 137 | 2 | 6,1 | 19321 | 139 | 2 | 2,1 | 22201 | 149 | 2 | 3,1 |
| 22801 | 151 | 2 | 12,1 | 24649 | 157 | 2 | 6,1 | 26569 | 163 | 2 | 11,1 |
| 27889 | 167 | 2 | 5,1 | 29929 | 173 | 2 | 5,1 | 32041 | 179 | 2 | 7,1 |
| 32761 | 181 | 2 | 18,1 | | | | | | | | |

Table 8: Table of $\gamma_1, \gamma_2$ for RBIBD$(5q + 1, 6, 1)$ construction

| $q$ | $\omega$ | $\gamma_1, \gamma_2$ | $q$ | $\omega$ | $\gamma_1, \gamma_2$ | $q$ | $\omega$ | $\gamma_1, \gamma_2$ | $q$ | $\omega$ | $\gamma_1, \gamma_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5041 | * | 139, 2672 | | | | | | | | | |
| 5077 | 2 | 2, 4951 | 5101 | 6 | 4912, 4634 | 5113 | 19 | 3542, 4393 | 5197 | 7 | 1330, 395 |
| 5209 | 17 | 2102, 2857 | 5233 | 10 | 2552, 1021 | 5281 | 7 | 4898, 3922 | 5329 | * | 1184, 1846 |
| 5413 | 5 | 3052, 2318 | 5437 | 5 | 116, 349 | 5449 | 7 | 760, 4034 | 5521 | 11 | 3692, 5446 |
| 5557 | 2 | 1, 4127 | 5569 | 13 | 4372, 5024 | 5581 | 6 | 586, 4997 | 5641 | 14 | 3910, 2897 |
| 5653 | 5 | 2926, 2486 | 5689 | 11 | 136, 4397 | 5701 | 2 | 1, 5456 | 5737 | 5 | 1777, 1817 |
| 5749 | 2 | 1, 3920 | 5821 | 6 | 3136, 1442 | 5857 | 7 | 4546, 1934 | 5869 | 2 | 1, 4907 |
| 5881 | 31 | 1108, 128 | 5953 | 7 | 1364, 1807 | 6037 | 5 | 266, 3574 | 6073 | 10 | 3622, 3260 |
| 6121 | 7 | 124, 5162 | 6133 | 5 | 3518, 2134 | 6217 | 5 | 1, 3365 | 6229 | 2 | 1, 2813 |
| 6241 | * | 320, 6106 | 6277 | 2 | 1, 662 | 6301 | 10 | 5722, 5495 | 6337 | 10 | 418, 4532 |
| 6361 | 19 | 4796, 3673 | 6373 | 2 | 1, 3776 | 6397 | 2 | 1, 5075 | 6421 | 6 | 760, 4985 |
| 6469 | 2 | 1, 1790 | 6481 | 7 | 2272, 1670 | 6529 | 7 | 5312, 4423 | 6553 | 10 | 392, 6151 |
| 6577 | 5 | 974, 1021 | 6637 | 2 | 2, 6337 | 6661 | 6 | 5164, 6098 | 6673 | 5 | 1, 4238 |
| 6709 | 2 | 2, 5383 | 6733 | 2 | 1, 32 | 6781 | 2 | 1, 6284 | 6793 | 10 | 2428, 782 |
| 6829 | 2 | 1, 4112 | 6841 | 22 | 5764, 1853 | 6889 | * | 1, 3449 | 6949 | 2 | 1, 6668 |
| 6961 | 13 | 6320, 2368 | 6997 | 5 | 2500, 2939 | 7057 | 5 | 1, 6080 | 7069 | 2 | 6493, 6512 |
| 7129 | 7 | 4108, 533 | 7177 | 10 | 1964, 4900 | 7213 | 5 | 1, 1289 | 7237 | 2 | 1, 7118 |
| 7297 | 5 | 2054, 3430 | 7309 | 6 | 4907, 5908 | 7321 | 7 | 5764, 6206 | 7333 | 6 | 3196, 7325 |
| 7369 | 7 | 1316, 535 | 7393 | 5 | 5768, 2344 | 7417 | 5 | 1490, 2236 | 7477 | 2 | 1, 3056 |
| 7489 | 7 | 1520, 2197 | 7537 | 7 | 1706, 1486 | 7549 | 2 | 1, 6839 | 7561 | 13 | 6556, 4190 |
| 7573 | 2 | 2, 6448 | 7621 | 2 | 1, 5738 | 7669 | 2 | 2, 1549 | 7681 | 17 | 3934, 89 |
| 7717 | 2 | 2, 4930 | 7741 | 7 | 1712, 3109 | 7753 | 10 | 1921, 6581 | 7789 | 2 | 1, 1610 |
| 7873 | 5 | 1, 4520 | 7921 | * | 1, 4049 | 7933 | 2 | 1, 6203 | 7993 | 5 | 6080, 7828 |
| 8017 | 5 | 1762, 5627 | 8053 | 2 | 2, 7291 | 8089 | 17 | 604, 2750 | 8101 | 6 | 3970, 32 |
| 8161 | 7 | 6454, 1721 | 8209 | 7 | 4732, 7640 | 8221 | 2 | 2, 7999 | 8233 | 10 | 6032, 2938 |
| 8269 | 2 | 1, 4148 | 8293 | 2 | 1, 2564 | 8317 | 6 | 406, 2831 | 8329 | 7 | 4246, 5768 |
| 8353 | 5 | 1, 155 | 8377 | 5 | 199, 4220 | 8389 | 6 | 3154, 2429 | 8461 | 6 | 1490, 7615 |
| 8521 | 13 | 5282, 4522 | 8581 | 6 | 7358, 7180 | 8629 | 6 | 5146, 2231 | 8641 | 17 | 6502, 5798 |
| 8677 | 2 | 2, 4462 | 8689 | 13 | 8086, 3332 | 8713 | 5 | 1, 4457 | 8737 | 5 | 4924, 7364 |
| 8761 | 23 | 3862, 8291 | 8821 | 2 | 1, 7016 | 8893 | 5 | 1, 296 | 8929 | 11 | 4030, 6302 |
| 8941 | 6 | 934, 3845 | 9001 | 7 | 550, 5153 | 9013 | 5 | 5744, 7588 | 9049 | 7 | 4762, 5168 |
| 9109 | 10 | 1294, 5339 | 9133 | 6 | 7312, 5465 | 9157 | 6 | 3038, 7204 | 9181 | 2 | 2, 1078 |
| 9241 | 13 | 1724, 6877 | 9277 | 5 | 1, 2627 | 9337 | 5 | 3788, 664 | 9349 | 2 | 1, 2396 |
| 9397 | 2 | 1, 8720 | 9409 | * | 3332, 4939 | 9421 | 2 | 2, 2899 | 9433 | 5 | 9386, 9103 |
| 9601 | 13 | 4052, 8371 | 9613 | 2 | 2, 8749 | 9649 | 7 | 5240, 7084 | 9661 | 2 | 1, 8888 |
| 9697 | 10 | 2908, 911 | 9721 | 7 | 7846, 872 | 9733 | 2 | 1, 5981 | 9769 | 13 | 1600, 6401 |
| 9781 | 6 | 1306, 470 | 9817 | 5 | 9091, 3092 | 9829 | 10 | 8407, 8687 | 9901 | 2 | 2, 7246 |
| 9949 | 2 | 2, 3502 | 9973 | 11 | 9905, 6202 | 10009 | 11 | 7316, 1003 | 10069 | 2 | 1, 7385 |
| 10093 | 2 | 1, 4121 | 10141 | 2 | 1, 5819 | 10177 | 7 | 7840, 7553 | 10201 | * | 1, 5201 |
| 10273 | 10 | 7220, 6955 | 10321 | 7 | 6868, 9698 | 10333 | 5 | 6808, 5699 | 10357 | 2 | 2, 8767 |
| 10369 | 13 | 1288, 4556 | 10429 | 7 | 5230, 4388 | 10453 | 5 | 325, 2 | 10477 | 2 | 2, 9613 |

Table 8: Table of $\gamma_1, \gamma_2$ for RBIBD$(5q+1, 6, 1)$ construction (cont.)

| $q$ | $\omega$ | $\gamma_1, \gamma_2$ | $q$ | $\omega$ | $\gamma_1, \gamma_2$ | $q$ | $\omega$ | $\gamma_1, \gamma_2$ | $q$ | $\omega$ | $\gamma_1, \gamma_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10501 | 2 | 1, 1781 | 10513 | 7 | 10300, 4175 | 10597 | 5 | 1, 8246 | 10609 | * | 9152, 3670 |
| 10657 | 7 | 832, 3224 | 10729 | 7 | 10004, 7402 | 10753 | 11 | 1526, 4051 | 10789 | 2 | 1, 10187 |
| 10837 | 2 | 1, 3503 | 10861 | 2 | 1, 3776 | 10909 | 2 | 1, 8576 | 10957 | 5 | 1, 5375 |
| 10993 | 7 | 8420, 7621 | 11113 | 13 | 10982, 10129 | 11149 | 10 | 3316, 5636 | 11161 | 7 | 550, 1244 |
| 11173 | 5 | 1, 4388 | 11197 | 2 | 2, 8641 | 11257 | 10 | 3272, 3517 | 11317 | 2 | 1, 10364 |
| 11329 | 7 | 6988, 5372 | 11353 | 7 | 10726, 9839 | 11437 | 2 | 1, 6539 | 11449 | * | 1, 6374 |
| 11497 | 7 | 830, 4138 | 11593 | 5 | 6526, 1427 | 11617 | 10 | 6992, 9850 | 11677 | 2 | 2, 3949 |
| 11689 | 7 | 9446, 9679 | 11701 | 6 | 2165, 7399 | 11821 | 2 | 2023, 2054 | 11833 | 5 | 8684, 5908 |
| 11881 | * | 8360, 8950 | 11941 | 10 | 5575, 10760 | 11953 | 5 | 1, 8366 | 12037 | 5 | 3043, 659 |
| 12049 | 13 | 10048, 4532 | 12073 | 7 | 1480, 3446 | 12097 | 5 | 4756, 11165 | 12109 | 6 | 1156, 9107 |
| 12157 | 2 | 1, 4187 | 12241 | 7 | 1994, 9940 | 12253 | 2 | 1, 4187 | 12277 | 2 | 1, 10226 |
| 12289 | 11 | 4172, 2470 | 12301 | 2 | 1, 10916 | 12373 | 2 | 1, 1193 | 12409 | 7 | 7448, 10492 |
| 12421 | 7 | 3604, 10133 | 12433 | 13 | 6362, 8014 | 12457 | 10 | 3400, 7106 | 12517 | 6 | 2404, 5555 |
| 12541 | 14 | 4735, 8225 | 12553 | 5 | 7852, 7853 | 12577 | 10 | 4880, 12298 | 12589 | 2 | 1, 10634 |
| 12601 | 11 | 10718, 2314 | 12613 | 2 | 1, 12092 | 12637 | 2 | 1, 1064 | 12697 | 7 | 2468, 2143 |
| 12721 | 13 | 1570, 7628 | 12757 | 2 | 2, 7567 | 12769 | * | 1, 8783 | 12781 | 2 | 1, 1100 |
| 12829 | 2 | 1, 662 | 12841 | 21 | 9590, 418 | 12853 | 5 | 1, 5402 | 12889 | 13 | 9982, 8822 |
| 12973 | 14 | 1, 2519 | 13009 | 7 | 2582, 9454 | 13033 | 5 | 5006, 9103 | 13093 | 6 | 7162, 5744 |
| 13177 | 5 | 1, 3584 | 13249 | 7 | 11582, 11584 | 13297 | 5 | 12055, 10058 | 13309 | 6 | 12670, 4796 |
| 13381 | 10 | 7855, 4640 | 13417 | 5 | 10640, 7327 | 13441 | 11 | 8728, 4601 | 13477 | 2 | 2, 9088 |
| 13513 | 5 | 6772, 3707 | 13537 | 7 | 9542, 7150 | 13597 | 5 | 2756, 2170 | 13633 | 5 | 12112, 12539 |
| 13669 | 6 | 1, 335 | 13681 | 22 | 12352, 11822 | 13693 | 6 | 6652, 1856 | 13729 | 23 | 13240, 9743 |
| 13789 | 7 | 10700, 11728 | 13873 | 5 | 3838, 11099 | 13921 | 7 | 316, 1814 | 13933 | 2 | 1, 5 |
| 14029 | 6 | 766, 6005 | 14149 | 6 | 11944, 10304 | 14173 | 2 | 2, 3319 | 14197 | 11 | 12994, 3098 |
| 14221 | 2 | 7951, 8267 | 14281 | 19 | 12674, 4423 | 14293 | 6 | 1468, 10805 | 14341 | 2 | 2, 3193 |
| 14389 | 2 | 1, 2672 | 14401 | 11 | 10264, 10265 | 14437 | 5 | 10028, 1018 | 14449 | 22 | 3956, 1 |
| 14461 | 2 | 1, 14270 | 14533 | 2 | 2, 1957 | 14557 | 2 | 2, 8212 | 14593 | 5 | 1, 10751 |
| 14629 | 2 | 13699, 1328 | 14641 | * | 1, 5909 | 14653 | 2 | 1, 10595 | 14713 | 5 | 13996, 11606 |
| 14737 | 10 | 1588, 13580 | 14797 | 2 | 1, 263 | 14821 | 2 | 2, 7120 | 14869 | 2 | 1, 8693 |
| 14929 | 7 | 11116, 6950 | 15013 | 2 | 1, 4772 | 15061 | 2 | 1, 5276 | 15073 | 5 | 1, 1718 |
| 15121 | 11 | 952, 953 | 15193 | 5 | 1, 13289 | 15217 | 10 | 12508, 13388 | 15241 | 11 | 11120, 11860 |
| 15277 | 6 | 14104, 2312 | 15289 | 11 | 2422, 2423 | 15313 | 5 | 13384, 3734 | 15349 | 2 | 1, 9620 |
| 15361 | 7 | 2696, 10456 | 15373 | 2 | 1, 3191 | 15493 | 5 | 682, 11783 | 15541 | 6 | 11810, 11059 |
| 15601 | 23 | 14680, 1067 | 15625 | * | 1, 2966 | 15649 | 11 | 2720, 13648 | 15661 | 2 | 1, 6683 |
| 15733 | 6 | 12778, 5399 | 15817 | 5 | 5908, 8213 | 15877 | 5 | 10826, 13150 | 15889 | 21 | 1046, 11086 |
| 15901 | 10 | 7934, 15874 | 15913 | 5 | 10399, 15884 | 15937 | 7 | 6034, 14057 | 15973 | 7 | 6202, 15356 |
| 16033 | 5 | 11842, 12152 | 16057 | 7 | 14072, 13192 | 16069 | 2 | 1, 11744 | 16129 | * | 9344, 14698 |
| 16141 | 6 | 8495, 2323 | 16189 | 2 | 1, 7151 | 16249 | 17 | 1552, 617 | 16273 | 7 | 4628, 14185 |
| 16333 | 2 | 1, 2051 | 16369 | 7 | 2384, 7294 | 16381 | 2 | 1, 2777 | 16417 | 10 | 12104, 10915 |
| 16453 | 2 | 1, 10799 | 16477 | 2 | 2, 9283 | 16561 | 7 | 14656, 6188 | 16573 | 2 | 2, 6043 |
| 16633 | 15 | 5080, 8360 | 16657 | 5 | 9163, 5075 | 16693 | 2 | 2, 11515 | 16729 | 13 | 4898, 10468 |
| 16741 | 6 | 8080, 5459 | 16921 | 17 | 12920, 14473 | 16981 | 2 | 1, 12800 | 16993 | 10 | 1474, 11753 |
| 17029 | 10 | 11056, 8372 | 17041 | 7 | 11060, 14719 | 17053 | 2 | 2, 3835 | 17077 | 2 | 1, 12101 |
| 17137 | 5 | 16760, 12640 | 17161 | * | 1, 11225 | 17209 | 14 | 12448, 4949 | 17257 | 5 | 7484, 7450 |
| 17293 | 7 | 13666, 13463 | 17317 | 2 | 2, 13246 | 17341 | 6 | 11932, 12620 | 17377 | 7 | 8428, 7226 |
| 17389 | 2 | 1, 2498 | 17401 | 11 | 2228, 5983 | 17449 | 14 | 15778, 2732 | 17497 | 5 | 13184, 5302 |
| 17509 | 2 | 1, 8018 | 17569 | 11 | 15652, 16250 | 17581 | 10 | 3955, 10442 | 17713 | 7 | 15436, 12746 |
| 17737 | 7 | 4792, 2837 | 17749 | 2 | 2, 2785 | 17761 | 19 | 17426, 2638 | 17881 | 7 | 2056, 3233 |
| 17929 | 11 | 16508, 2962 | 17977 | 5 | 2812, 4379 | 17989 | 2 | 1, 998 | 18013 | 2 | 1, 9518 |
| 18049 | 13 | 10544, 12631 | 18061 | 6 | 2146, 5492 | 18097 | 5 | 14114, 2890 | 18121 | 23 | 4336, 13016 |
| 18133 | 5 | 1, 12989 | 18169 | 11 | 3140, 6952 | 18181 | 2 | 1, 1661 | 18217 | 7 | 14410, 15248 |
| 18229 | 2 | 2, 4819 | 18253 | 5 | 14860, 12986 | 18289 | 13 | 14810, 6064 | 18301 | 6 | 1462, 8693 |
| 18313 | 10 | 15884, 7108 | 18397 | 6 | 3568, 3569 | 18433 | 5 | 6296, 826 | 18457 | 5 | 9572, 6664 |
| 18481 | 13 | 3824, 1474 | 18493 | 2 | 2, 10612 | 18517 | 6 | 16918, 18227 | 18541 | 6 | 10774, 10541 |
| 18553 | 5 | 15734, 17341 | 18637 | 2 | 2, 10984 | 18661 | 10 | 2140, 455 | 18757 | 2 | 1, 3233 |
| 18769 | * | 1, 6425 | 18793 | 5 | 1, 12797 | 18913 | 7 | 9290, 10456 | 18973 | 2 | 1, 18872 |
| 19009 | 23 | 9100, 17702 | 19069 | 2 | 1, 13226 | 19081 | 17 | 10144, 13082 | 19141 | 2 | 1, 15572 |
| 19213 | 5 | 16090, 6335 | 19237 | 2 | 1, 18398 | 19249 | 7 | 5728, 7874 | 19273 | 5 | 18127, 9122 |
| 19309 | 6 | 3886, 2162 | 19321 | * | 140, 17155 | 19333 | 2 | 1, 13478 | 19381 | 7 | 5315, 5989 |
| 19417 | 5 | 1, 197 | 19429 | 6 | 11350, 10025 | 19441 | 13 | 4264, 17684 | 19477 | 6 | 12616, 13475 |
| 19489 | 19 | 15676, 1226 | 19501 | 2 | 1, 18218 | 19597 | 2 | 1, 5942 | 19609 | 13 | 3680, 16945 |
| 19681 | 11 | 7420, 1004 | 19717 | 2 | 1, 11135 | 19753 | 5 | 1, 11984 | 19777 | 11 | 1, 6197 |
| 19801 | 13 | 844, 15794 | 19813 | 2 | 2, 7 | 19861 | 11 | 2972, 5146 | 19993 | 10 | 9028, 7298 |
| 20029 | 2 | 1, 2834 | 20089 | 7 | 9188, 10291 | 20101 | 6 | 3382, 13340 | 20113 | 10 | 11512, 5825 |
| 20149 | 2 | 2209, 4421 | 20161 | 13 | 19286, 4951 | 20173 | 2 | 2, 13003 | 20233 | 5 | 956, 9229 |
| 20269 | 2 | 2, 14377 | 20341 | 2 | 1, 11054 | 20353 | 5 | 15032, 11791 | 20389 | 6 | 5428, 9485 |
| 20509 | 2 | 1, 13403 | 20521 | 11 | 9176, 2035 | 20533 | 2 | 2, 49 | 20593 | 5 | 13255, 11429 |
| 20641 | 7 | 11768, 14755 | 20749 | 2 | 2, 13165 | 20773 | 2 | 2, 5089 | 20809 | 7 | 9512, 12859 |
| 20857 | 10 | 19276, 2 | 20929 | 7 | 12710, 8773 | 21001 | 11 | 8192, 2038 | 21013 | 2 | 1, 17255 |
| 21061 | 7 | 15310, 13976 | 21121 | 19 | 1706, 4267 | 21157 | 2 | 2, 12016 | 21169 | 13 | 18290, 9988 |
| 21193 | 11 | 20638, 15902 | 21277 | 6 | 7174, 14924 | 21313 | 5 | 10700, 17416 | 21397 | 2 | 2, 4144 |
| 21433 | 5 | 5272, 1958 | 21481 | 13 | 1, 8999 | 21493 | 2 | 1, 10301 | 21517 | 5 | 1, 6440 |
| 21529 | 11 | 10156, 3029 | 21577 | 5 | 18757, 15350 | 21589 | 2 | 2, 3031 | 21601 | 7 | 4060, 20948 |
| 21613 | 2 | 1, 2357 | 21649 | 14 | 19786, 5327 | 21661 | 2 | 1, 461 | 21673 | 10 | 14456, 13774 |
| 21757 | 5 | 14581, 12776 | 21817 | 7 | 12268, 10223 | 21841 | 11 | 20860, 6203 | 21937 | 7 | 18436, 18515 |
| 21961 | 17 | 11968, 20306 | 21997 | 7 | 17150, 3709 | 22093 | 6 | 13552, 17411 | 22129 | 19 | 4874, 18760 |
| 22153 | 5 | 3850, 1205 | 22189 | 2 | 1, 986 | 22201 | * | 1, 11249 | 22273 | 5 | 596, 2845 |
| 22369 | 11 | 1366, 10478 | 22381 | 10 | 16025, 13972 | 22441 | 14 | 1718, 13489 | 22453 | 5 | 5140, 9476 |
| 22501 | 2 | 1, 20270 | 22549 | 2 | 1, 13214 | 22573 | 6 | 11153, 15028 | 22621 | 2 | 1, 7058 |
| 22669 | 2 | 2, 15937 | 22717 | 2 | 2, 11935 | 22741 | 7 | 16178, 16282 | 22777 | 7 | 15584, 12658 |
| 22801 | * | 6080, 5050 | 22861 | 2 | 2, 6448 | 22921 | 7 | 17522, 17704 | 22993 | 5 | 18682, 12599 |
| 23017 | 5 | 1, 11090 | 23029 | 2 | 1, 1295 | 23041 | 11 | 19196, 14560 | 23053 | 2 | 2, 18520 |
| 23173 | 5 | 1, 15995 | 23197 | 2 | 2, 19930 | 23209 | 31 | 21868, 16094 | 23269 | 6 | 1018, 2183 |
| 23293 | 5 | 22196, 9883 | 23473 | 5 | 1069, 18710 | 23497 | 5 | 1162, 15242 | 23509 | 2 | 1, 8624 |
| 23557 | 5 | 7423, 12947 | 23581 | 6 | 22840, 21416 | 23593 | 5 | 3658, 12836 | 23629 | 2 | 1, 1244 |
| 23677 | 5 | 9142, 4175 | 23689 | 11 | 5786, 2950 | 23761 | 7 | 10520, 11980 | 23773 | 5 | 1, 15002 |
| 23833 | 5 | 10393, 16862 | 23857 | 5 | 1072, 19442 | 23869 | 2 | 2707, 13469 | 23893 | 5 | 6061, 7046 |
| 23917 | 2 | 1, 4331 | 23929 | 7 | 12862, 22823 | 23977 | 5 | 1, 16976 | 24001 | 14 | 7892, 8425 |
| 24049 | 19 | 5878, 20237 | 24061 | 10 | 7622, 655 | 24097 | 5 | 3206, 20662 | 24109 | 2 | 2, 23602 |
| 24121 | 13 | 9304, 17294 | 24133 | 6 | 2344, 16832 | 24169 | 11 | 2710, 2675 | 24181 | 17 | 17026, 17621 |
| 24229 | 2 | 1, 4931 | 24337 | 5 | 21209, 7693 | 24373 | 7 | 6071, 7885 | 24421 | 7 | 20561, 8242 |
| 24469 | 14 | 22360, 15536 | 24481 | 11 | 15412, 13511 | 24517 | 5 | 21448, 10313 | 24649 | * | 1106, 6313 |
| 24697 | 5 | 15332, 6724 | 24709 | 2 | 1, 8516 | 24733 | 2 | 2, 21301 | 24781 | 2 | 1, 11570 |
| 24793 | 5 | 2044, 16271 | 24841 | 14 | 20692, 11495 | 24877 | 5 | 1, 10679 | 24889 | 11 | 21154, 1313 |
| 25033 | 5 | 11612, 3961 | 25057 | 5 | 18556, 2558 | 25117 | 5 | 2899, 5405 | 25153 | 10 | 24922, 6530 |
| 25189 | 2 | 1, 3131 | 25237 | 2 | 2, 23773 | 25261 | 7 | 9092, 12772 | 25309 | 13 | 1369, 19229 |
| 25321 | 19 | 19334, 17353 | 25357 | 2 | 1, 2330 | 25453 | 2 | 1, 17276 | 25537 | 10 | 18320, 4567 |
| 25561 | 11 | 9550, 15542 | 25609 | 7 | 6556, 23363 | 25621 | 10 | 6478, 19172 | 25633 | 5 | 15532, 21185 |
| 25657 | 5 | 1, 20807 | 25693 | 2 | 1, 25007 | 25717 | 2 | 1, 98 | 25741 | 6 | 17596, 3233 |
| 25801 | 7 | 16388, 22261 | 25849 | 7 | 19568, 9229 | 25873 | 10 | 13210, 22250 | 25933 | 2 | 1, 20000 |
| 25969 | 7 | 18556, 18518 | 25981 | 11 | 1, 14681 | 26017 | 5 | 15388, 1172 | 26029 | 6 | 15934, 5747 |
| 26041 | 13 | 6356, 13774 | 26053 | 2 | 1, 13421 | 26113 | 7 | 404, 9238 | 26161 | 13 | 20210, 9586 |
| 26209 | 11 | 23402, 5212 | 26293 | 6 | 1546, 7430 | 26317 | 6 | 22060, 18707 | 26437 | 5 | 1, 12968 |
| 26449 | 7 | 11926, 4547 | 26497 | 5 | 1, 13922 | 26557 | 2 | 1, 1226 | 26569 | * | 21484, 26120 |
| 26641 | 7 | 8024, 23239 | 26701 | 22 | 15712, 13418 | 26713 | 10 | 18358, 9077 | 26737 | 10 | 22966, 19802 |
| 26821 | 2 | 2, 6376 | 26833 | 5 | 25156, 4691 | 26881 | 11 | 26152, 17867 | 26893 | 5 | 1, 290 |
| 26953 | 7 | 21808, 1400 | 27061 | 2 | 1, 4259 | 27073 | 5 | 8692, 25346 | 27109 | 7 | 982, 14381 |
| 27241 | 17 | 16580, 1912 | 27253 | 2 | 2, 8710 | 27277 | 6 | 2284, 21938 | 27337 | 5 | 12754, 698 |
| 27361 | 7 | 21146, 17236 | 27397 | 2 | 1, 2891 | 27409 | 13 | 22234, 2438 | 27457 | 7 | 8776, 9440 |
| 27481 | 7 | 17672, 8440 | 27529 | 7 | 8462, 26809 | 27541 | 19 | 7064, 26839 | 27673 | 11 | 26026, 12506 |
| 27697 | 5 | 1, 27392 | 27733 | 2 | 1, 9989 | 27793 | 5 | 2662, 2663 | 27817 | 5 | 1, 16328 |
| 27889 | * | 1, 9080 | 27901 | 2 | 1, 22772 | 27961 | 13 | 25168, 14954 | 27997 | 5 | 16556, 13741 |

Table 8: Table of $\gamma_1, \gamma_2$ for RBIBD$(5q+1, 6, 1)$ construction (cont.)

| $q$ | $\omega$ | $\gamma_1, \gamma_2$ | $q$ | $\omega$ | $\gamma_1, \gamma_2$ | $q$ | $\omega$ | $\gamma_1, \gamma_2$ | $q$ | $\omega$ | $\gamma_1, \gamma_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 28057 | 5 | 1, 19916 | 28069 | 7 | 7490, 10606 | 28081 | 19 | 2258, 5971 | 28201 | 11 | 11596, 27434 |
| 28297 | 5 | 1, 4175 | 28309 | 2 | 1, 22127 | 28393 | 15 | 1540, 20747 | 28429 | 2 | 1, 3530 |
| 28477 | 2 | 2, 24520 | 28513 | 5 | 22136, 16180 | 28537 | 5 | 27301, 2339 | 28549 | 2 | 1, 26345 |
| 28561 | * | 11900, 17422 | 28573 | 2 | 2, 25849 | 28597 | 2 | 2, 14152 | 28621 | 13 | 13390, 2225 |
| 28657 | 5 | 5584, 27263 | 28669 | 6 | 22144, 28178 | 28729 | 22 | 5704, 17360 | 28753 | 10 | 21868, 10718 |
| 28789 | 7 | 23095, 18800 | 28813 | 2 | 2, 3892 | 28837 | 2 | 1, 25505 | 28909 | 2 | 26773, 19544 |
| 28921 | 11 | 4856, 1852 | 28933 | 2 | 1, 9656 | 29017 | 5 | 1, 11276 | 29077 | 2 | 1, 5912 |
| 29101 | 2 | 2, 16819 | 29137 | 5 | 2180, 7261 | 29173 | 2 | 2, 19000 | 29209 | 7 | 15704, 17776 |
| 29221 | 2 | 1, 22049 | 29269 | 6 | 10870, 2 | 29389 | 2 | 1, 22643 | 29401 | 13 | 25006, 26942 |
| 29437 | 2 | 1, 11222 | 29473 | 5 | 15812, 22645 | 29569 | 17 | 27170, 17158 | 29581 | 10 | 18955, 4652 |
| 29629 | 7 | 23501, 6397 | 29641 | 7 | 1718, 20311 | 29761 | 17 | 12880, 18377 | 29833 | 5 | 14360, 19414 |
| 29881 | 7 | 4586, 24682 | 29917 | 2 | 1, 14357 | 29929 | * | 1, 22928 | 29989 | 2 | 2, 26377 |
| 30013 | 2 | 1, 18311 | 30097 | 10 | 12164, 29641 | 30109 | 2 | 1, 28775 | 30133 | 5 | 27092, 23647 |
| 30169 | 7 | 17450, 10954 | 30181 | 2 | 5329, 29351 | 30241 | 11 | 10472, 25018 | 30253 | 2 | 1, 10823 |
| 30313 | 5 | 21214, 12914 | 30469 | 2 | 1, 16979 | 30493 | 6 | 27863, 29215 | 30517 | 2 | 1, 15788 |
| 30529 | 13 | 5612, 28000 | 30553 | 5 | 13934, 7789 | 30577 | 5 | 1, 2690 | 30637 | 2 | 1, 14846 |
| 30649 | 7 | 11572, 6134 | 30661 | 2 | 1, 20540 | 30697 | 10 | 18512, 5626 | 30757 | 5 | 25480, 6938 |
| 30781 | 2 | 1, 7949 | 30817 | 5 | 14228, 15202 | 30829 | 2 | 1, 21455 | 30841 | 7 | 3944, 12718 |
| 30853 | 2 | 1, 3725 | 30937 | 15 | 20056, 1730 | 30949 | 10 | 27802, 9560 | 31033 | 10 | 1496, 27679 |
| 31069 | 2 | 1, 15086 | 31081 | 13 | 22708, 28232 | 31153 | 10 | 17116, 4904 | 31177 | 7 | 10100, 26794 |
| 31189 | 13 | 3484, 5732 | 31237 | 6 | 16054, 11132 | 31249 | 23 | 20, 27442 | 31321 | 7 | 2692, 11633 |
| 31333 | 5 | 30956, 14797 | 31357 | 2 | 1, 5069 | 31393 | 5 | 15572, 13420 | 31477 | 6 | 6286, 18374 |
| 31489 | 7 | 8638, 13253 | 31513 | 7 | 31180, 16577 | 31573 | 5 | 29054, 2242 | 31657 | 5 | 1, 18815 |
| 31729 | 7 | 21548, 16459 | 31741 | 6 | 30268, 16931 | 31849 | 14 | 30238, 26933 | 31873 | 11 | 5260, 15536 |
| 31957 | 2 | 1, 4562 | 31981 | 6 | 8270, 15340 | 32029 | 2 | 13717, 9266 | 32041 | * | 1, 29984 |
| 32077 | 2 | 2, 3400 | 32089 | 13 | 344, 7162 | 32173 | 5 | 11794, 18578 | 32233 | 5 | 21944, 4837 |
| 32257 | 15 | 15754, 9191 | 32341 | 2 | 2, 27484 | 32353 | 15 | 31364, 8821 | 32377 | 5 | 14768, 32086 |
| 32401 | 7 | 10090, 6638 | 32413 | 5 | 15232, 10499 | 32497 | 7 | 17752, 14936 | 32533 | 2 | 1, 5 |
| 32569 | 7 | 19490, 10921 | 32653 | 2 | 1, 26084 | 32713 | 5 | 18577, 8342 | 32749 | 2 | 1, 26279 |
| 32761 | * | 24934, 30617 | | | | | | | | | |

Table 9: Table of $m, w, c, c', c''$ for BIBD$(q, 13, 1)$ construction

| $q$ | $m$ | $w$ | c | c' | c" | $q$ | $m$ | $w$ | c | c' | c" |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6241 | 1430 | 23 | 5276 | 5389 | 3429 | 8737 | 1976 | 2268 | 2107 | 4412 | 8312 |
| 9829 | 6740 | 748 | 3942 | 3381 | 5212 | 14197 | 6084 | 5123 | 12586 | 6980 | 6438 |
| 15601 | 4157 | 3358 | 14688 | 10476 | 4395 | 16069 | 4720 | 14117 | 7813 | 1673 | 2810 |
| 16381 | 12288 | 3393 | 4508 | 6606 | 9926 | 16693 | 3004 | 5153 | 9988 | 14869 | 13051 |
| 18097 | 2895 | 12109 | 10205 | 1950 | 3646 | 19813 | 2233 | 7510 | 9356 | 1569 | 13237 |
| 20593 | 1341 | 143 | 1651 | 10214 | 6166 | 20749 | 6598 | 3119 | 3006 | 4800 | 9239 |
| 21061 | 20993 | 3375 | 12210 | 152 | 5448 | 21529 | 14249 | 3972 | 2863 | 19892 | 16577 |
| 21841 | 9084 | 8529 | 20542 | 16054 | 657 | 21997 | 3809 | 11605 | 16088 | 20775 | 20879 |
| 22153 | 8119 | 10588 | 15483 | 6187 | 16321 | 22621 | 14978 | 13088 | 20440 | 17145 | 22198 |
| 22777 | 14977 | 1740 | 8251 | 14704 | 20285 | 23557 | 9945 | 22598 | 21005 | 2174 | 19905 |
| 23869 | 13105 | 14848 | 20525 | 16395 | 21641 | 24181 | 5594 | 155 | 12600 | 13087 | 11990 |
| 24337 | 19291 | 12508 | 9143 | 22389 | 23346 | 25117 | 21357 | 274 | 16277 | 19854 | 9074 |
| 25741 | 7120 | 24210 | 24046 | 25039 | 24616 | 26053 | 22389 | 12522 | 24670 | 3452 | 9299 |
| 26209 | 16112 | 24180 | 21052 | 22252 | 7177 | 26833 | 6817 | 18598 | 18591 | 7119 | 21864 |
| 27457 | 4369 | 23868 | 296 | 4005 | 3179 | 28081 | 4962 | 1765 | 9644 | 21150 | 21612 |
| 28549 | 18714 | 7759 | 6616 | 18676 | 1579 | 29017 | 7565 | 1964 | 18357 | 4086 | 19246 |
| 29173 | 10964 | 9972 | 19507 | 20561 | 9983 | 29641 | 8581 | 19912 | 17148 | 29313 | 18919 |
| 30109 | 26012 | 20130 | 21915 | 29240 | 2354 | 30577 | 5687 | 28211 | 13065 | 20020 | 15930 |
| 31357 | 4884 | 10608 | 10496 | 13281 | 17897 | 31513 | 13169 | 10563 | 4205 | 25316 | 16700 |
| 31981 | 5015 | 12636 | 12148 | 21722 | 25623 | | | | | | |

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD $13^q$ construction

| $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ | $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ |
|---|---|---|---|---|---|
| 13 | 2 | 2, 1, 4, 5, 9 | 121 | * | 4, 73, 98, 119, 15 |
| 157 | 5 | 20, 1, 40, 59, 117 | 229 | 6 | 190, 209, 152, 133, 171 |
| 277 | 5 | 218, 247, 40, 149, 207 | 361 | * | 305, 4, 116, 331, 57 |
| 397 | 5 | 61, 23, 320, 358, 297 | 529 | * | 212, 189, 376, 377, 517 |
| 601 | 7 | 223, 155, 52, 291, 2 | 613 | 2 | 92, 337, 184, 245, 459 |
| 661 | 2 | 634, 197, 32, 469, 495 | 709 | 2 | 236, 649, 118, 59, 177 |
| 733 | 6 | 602, 667, 484, 53, 549 | 757 | 2 | 236, 307, 496, 527, 87 |
| 829 | 2 | 496, 152, 261, 557, 583 | 841 | * | 741, 305, 452, 838, 553 |
| 853 | 2 | 155, 200, 193, 334, 393 | 877 | 2 | 765, 734, 167, 718, 607 |
| 937 | 5 | 497, 904, 307, 771, 836 | 961 | * | 787, 376, 543, 767, 704 |
| 997 | 7 | 586, 55, 9, 923, 944 | 1009 | 11 | 898, 269, 512, 561, 235 |
| 1021 | 10 | 302, 952, 1, 497, 525 | 1033 | 5 | 803, 393, 446, 904, 565 |
| 1069 | 6 | 401, 109, 380, 622, 489 | 1093 | 5 | 527, 358, 800, 783, 1009 |
| 1117 | 2 | 885, 299, 340, 1070, 1093 | 1129 | 11 | 856, 992, 573, 437, 709 |
| 1153 | 5 | 403, 953, 358, 3, 554 | 1201 | 11 | 215, 673, 416, 286, 939 |
| 1213 | 2 | 911, 760, 1100, 1179, 931 | 1237 | 2 | 1069, 725, 274, 1166, 669 |
| 1249 | 7 | 520, 375, 1013, 1040, 1 | 1297 | 10 | 971, 91, 8, 262, 783 |
| 1321 | 13 | 857, 796, 657, 20, 841 | 1369 | * | 1306, 853, 734, 443, 1035 |
| 1381 | 2 | 340, 1196, 751, 53, 585 | 1429 | 6 | 164, 227, 1246, 1185, 163 |
| 1453 | 2 | 947, 543, 1366, 973, 464 | 1489 | 14 | 1366, 1127, 1341, 31, 956 |
| 1549 | 2 | 821, 361, 1444, 927, 50 | 1597 | 11 | 28, 1310, 283, 1565, 1593 |
| 1609 | 7 | 1510, 1093, 1157, 513, 1358 | 1621 | 2 | 1167, 893, 1582, 691, 2 |
| 1657 | 11 | 460, 1370, 509, 1423, 1533 | 1669 | 2 | 952, 641, 1447, 609, 1586 |
| 1681 | * | 1244, 502, 561, 1193, 235 | 1693 | 2 | 61, 1409, 1305, 778, 104 |
| 1741 | 2 | 1060, 369, 134, 29, 1 | 1753 | 7 | 52, 781, 893, 1370, 327 |
| 1777 | 5 | 719, 1510, 867, 1154, 577 | 1789 | 6 | 1448, 1199, 814, 469, 651 |
| 1801 | 11 | 1037, 61, 382, 279, 1142 | 1849 | * | 1625, 525, 52, 1375, 440 |
| 1861 | 2 | 1175, 1531, 1840, 1454, 3 | 1873 | 10 | 4, 465, 1667, 1, 1844 |
| 1933 | 5 | 676, 1339, 1700, 11, 1329 | 1993 | 5 | 1819, 1858, 1385, 146, 1203 |
| 2017 | 5 | 875, 481, 297, 964, 44 | 2029 | 2 | 1959, 527, 1597, 1498, 2 |
| 2053 | 2 | 1660, 819, 623, 1369, 2 | 2089 | 7 | 275, 279, 1303, 190, 1442 |
| 2113 | 5 | 725, 1450, 806, 1485, 391 | 2137 | 10 | 634, 1441, 1329, 395, 1520 |
| 2161 | 23 | 1651, 837, 701, 760, 326 | 2197 | * | 1019, 1417, 1966, 470, 549 |
| 2209 | * | 1976, 1540, 179, 1903, 945 | 2221 | 2 | 784, 1484, 1031, 1881, 529 |
| 2269 | 2 | 1396, 763, 723, 731, 2 | 2281 | 7 | 1209, 815, 1186, 979, 38 |
| 2293 | 2 | 640, 759, 899, 1658, 1 | 2341 | 7 | 1966, 1199, 1465, 2126, 1215 |
| 2377 | 5 | 1714, 2084, 421, 719, 1719 | 2389 | 2 | 1948, 1621, 605, 1209, 2378 |
| 2401 | * | 2361, 1805, 1556, 1960, 1 | 2437 | 2 | 1180, 1519, 831, 1709, 2 |
| 2473 | 5 | 747, 745, 1942, 53, 656 | 2521 | 17 | 2275, 998, 389, 2284, 855 |
| 2557 | 2 | 1525, 2045, 2163, 232, 2 | 2593 | 7 | 657, 2494, 2587, 1391, 2048 |
| 2617 | 5 | 1883, 255, 157, 532, 1844 | 2677 | 2 | 1862, 352, 455, 2397, 1 |
| 2689 | 19 | 1699, 610, 305, 951, 1244 | 2713 | 5 | 1135, 1990, 1535, 471, 1916 |
| 2749 | 6 | 2504, 2399, 1414, 1075, 495 | 2797 | 2 | 2639, 1125, 1190, 1570, 1 |
| 2809 | * | 1157, 8, 249, 2806, 1 | 2833 | 5 | 1565, 2721, 1570, 206, 1 |
| 2857 | 11 | 345, 647, 883, 1234, 1598 | 2917 | 5 | 1300, 955, 881, 2282, 825 |

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD $13^q$ construction (cont.)

| $q$ | $\varpi$ | $\gamma_1, \ldots, \gamma_5$ | $q$ | $\varpi$ | $\gamma_1, \ldots, \gamma_5$ |
|---|---|---|---|---|---|
| 2953 | 13 | 73, 1382, 2920, 431, 2511 | 3001 | 14 | 598, 1712, 2765, 2995, 2967 |
| 3037 | 2 | 41, 2626, 411, 1472, 1 | 3049 | 11 | 1709, 1561, 2344, 2133, 2048 |
| 3061 | 6 | 1870, 2744, 113, 2299, 1815 | 3109 | 6 | 2575, 1072, 2777, 1167, 1694 |
| 3121 | 7 | 505, 2067, 1804, 1139, 1142 | 3169 | 7 | 425, 2439, 1555, 1624, 548 |
| 3181 | 7 | 2643, 928, 517, 71, 3062 | 3217 | 5 | 928, 2195, 368, 27, 1 |
| 3229 | 6 | 793, 1642, 2258, 1955, 93 | 3253 | 2 | 1999, 225, 2260, 2531, 2 |
| 3301 | 6 | 3027, 142, 2147, 1495, 866 | 3313 | 10 | 3153, 2113, 1582, 2525, 1688 |
| 3361 | 22 | 1888, 2565, 1651, 503, 1100 | 3373 | 5 | 1684, 2331, 689, 914, 1 |
| 3433 | 5 | 765, 1516, 1265, 1333, 2126 | 3457 | 7 | 2902, 95, 2839, 2660, 813 |
| 3469 | 2 | 3371, 116, 724, 939, 1 | 3481 | * | 2159, 260, 976, 759, 1 |
| 3517 | 2 | 3233, 3091, 1737, 1906, 2 | 3529 | 17 | 941, 2353, 1191, 2146, 1682 |
| 3541 | 7 | 3367, 430, 1190, 1223, 1035 | 3613 | 2 | 2217, 3160, 1937, 284, 1 |
| 3637 | 2 | 3371, 3388, 1634, 579, 1 | 3673 | 5 | 1101, 1409, 820, 619, 2024 |
| 3697 | 5 | 3262, 3439, 417, 1853, 1838 | 3709 | 2 | 3446, 1186, 593, 747, 1 |
| 3721 | * | 2133, 1451, 289, 1918, 1364 | 3733 | 2 | 121, 2981, 2122, 1923, 2 |
| 3769 | 7 | 2303, 2901, 2455, 3238, 878 | 3793 | 5 | 2045, 897, 409, 160, 1238 |
| 3853 | 2 | 1545, 16, 3559, 401, 2 | 3877 | 2 | 3755, 2404, 3098, 423, 1 |
| 3889 | 11 | 3653, 2392, 1735, 1311, 1046 | 4021 | 2 | 1318, 819, 2096, 3605, 1 |
| 4057 | 5 | 419, 3224, 1744, 717, 1945 | 4093 | 2 | 3886, 3715, 1997, 111, 2 |
| 4129 | 13 | 2236, 665, 3578, 601, 2223 | 4153 | 5 | 2615, 1557, 520, 3866, 1 |
| 4177 | 5 | 3892, 3525, 2288, 2183, 2545 | 4201 | 11 | 3723, 4168, 907, 227, 776 |
| 4261 | 2 | 2672, 4234, 1385, 669, 1 | 4273 | 5 | 3538, 3625, 2157, 2705, 680 |
| 4297 | 5 | 1945, 777, 796, 773, 992 | 4357 | 2 | 512, 1563, 2909, 1438, 1 |
| 4441 | 21 | 2807, 2311, 2061, 2740, 3356 | 4489 | * | 1901, 4126, 841, 4065, 476 |
| 4513 | 7 | 1438, 3674, 217, 1601, 2019 | 4549 | 6 | 2306, 2041, 520, 3083, 267 |
| 4561 | 11 | 217, 3994, 819, 173, 608 | 4597 | 5 | 1913, 4147, 3932, 1876, 1245 |
| 4621 | 2 | 2943, 1234, 1286, 341, 1 | 4657 | 15 | 3124, 3884, 911, 2479, 2715 |
| 4729 | 17 | 4438, 4141, 4295, 2288, 4359 | 4789 | 2 | 4035, 2026, 1862, 4349, 1 |
| 4801 | 7 | 4012, 4489, 335, 3681, 4796 | 4813 | 2 | 875, 4671, 1156, 1166, 1 |
| 4861 | 11 | 1864, 4124, 1907, 2383, 657 | 4909 | 6 | 3331, 802, 335, 4802, 3657 |
| 4933 | 2 | 3883, 1522, 3401, 1101, 2 | 4957 | 2 | 981, 1330, 37, 3563, 2 |
| 4969 | 11 | 2686, 4131, 4429, 131, 4172 | 4993 | 5 | 2823, 2770, 3953, 1736, 1 |
| 5041 | * | 4822, 3795, 1181, 4334, 139 | 5077 | 2 | 4583, 2935, 27, 2650, 2 |
| 5101 | 6 | 2450, 1, 2116, 3563, 189 | 5113 | 19 | 721, 3896, 928, 1565, 2973 |
| 5197 | 7 | 4589, 2065, 2800, 3752, 5175 | 5209 | 17 | 5165, 646, 5089, 1053, 2102 |
| 5233 | 10 | 3033, 2777, 4228, 4945, 2552 | 5281 | 7 | 891, 2875, 4702, 4889, 4898 |
| 5329 | * | 971, 3166, 1621, 4587, 1184 | 5413 | 5 | 1346, 1198, 1577, 4471, 4485 |
| 5437 | 5 | 148, 3341, 1129, 386, 327 | 5449 | 7 | 879, 187, 3676, 5387, 4034 |
| 5521 | 11 | 523, 4919, 4174, 1215, 3692 | 5557 | 2 | 170, 1402, 3423, 4067, 1 |
| 5569 | 13 | 2648, 59, 3562, 3675, 1 | 5581 | 6 | 5327, 1912, 1550, 481, 4995 |
| 5641 | 14 | 391, 4865, 1708, 5163, 2180 | 5653 | 5 | 350, 4090, 1535, 2479, 3117 |
| 5689 | 11 | 724, 3799, 5151, 5633, 482 | 5701 | 2 | 4726, 3740, 4553, 183, 1 |
| 5737 | 5 | 4876, 3344, 627, 5489, 1777 | 5749 | 2 | 1318, 5660, 2753, 3363, 1 |
| 5821 | 6 | 620, 1181, 718, 2071, 2685 | 5857 | 7 | 2485, 4163, 2806, 1923, 4034 |
| 5869 | 2 | 320, 923, 789, 1228, 1 | 5881 | 31 | 1213, 808, 5013, 4667, 5534 |
| 5953 | 7 | 2995, 2698, 5390, 611, 1743 | 6037 | 5 | 2428, 5357, 188, 277, 1485 |
| 6073 | 10 | 3363, 4559, 916, 1861, 5816 | 6121 | 7 | 5083, 4234, 5627, 171, 4790 |
| 6133 | 5 | 1271, 3889, 4778, 1558, 891 | 6217 | 5 | 2948, 4204, 5277, 4079, 1 |
| 6229 | 2 | 921, 3562, 3188, 1205, 1 | 6241 | * | 2890, 735, 3167, 1021, 320 |
| 6277 | 2 | 548, 4223, 4017, 556, 1 | 6301 | 10 | 1607, 3464, 4195, 2824, 1737 |
| 6337 | 10 | 5099, 1714, 465, 595, 836 | 6361 | 19 | 5242, 3291, 341, 4117, 4796 |
| 6373 | 2 | 944, 4, 2643, 2777, 1 | 6397 | 2 | 158, 5, 6081, 1936, 1 |
| 6421 | 6 | 4600, 1166, 4223, 691, 5661 | 6469 | 2 | 1538, 622, 2039, 1479, 1 |
| 6481 | 7 | 759, 3172, 3881, 2035, 1406 | 6529 | 7 | 5981, 69, 856, 5383, 5312 |
| 6553 | 10 | 4705, 2453, 5547, 1828, 392 | 6577 | 5 | 6317, 5991, 2995, 2950, 974 |
| 6637 | 2 | 1961, 6580, 4933, 5709, 2 | 6661 | 6 | 5944, 3668, 2323, 3521, 1497 |
| 6673 | 5 | 2757, 508, 1217, 536, 1 | 6709 | 2 | 6551, 3832, 277, 2403, 2 |
| 6733 | 2 | 878, 3573, 3077, 6010, 1 | 6781 | 2 | 5584, 4736, 3749, 477, 1 |
| 6793 | 10 | 2566, 1, 3389, 2613, 4856 | 6829 | 2 | 2517, 650, 4949, 5848, 1 |
| 6841 | 22 | 3197, 139, 5128, 4803, 5756 | 6889 | * | 2476, 2459, 5625, 1268, 1 |
| 6949 | 2 | 5757, 3866, 1708, 4661, 1 | 6961 | 13 | 1921, 508, 4733, 2421, 6320 |
| 6997 | 5 | 5111, 1324, 986, 1111, 855 | 7057 | 5 | 2266, 2351, 5234, 3627, 1 |
| 7069 | 2 | 1216, 4238, 6743, 6327, 6493 | 7129 | 7 | 5267, 2176, 3903, 1729, 878 |
| 7177 | 10 | 4397, 610, 5163, 6841, 1964 | 7213 | 5 | 5528, 6597, 5531, 7006, 1 |
| 7237 | 2 | 1070, 4276, 1463, 5817, 1 | 7297 | 5 | 1675, 4833, 893, 5980, 2054 |
| 7309 | 6 | 4473, 2506, 1295, 253, 2402 | 7321 | 7 | 99, 5555, 157, 3574, 3158 |
| 7333 | 6 | 6277, 4138, 4376, 521, 4137 | 7369 | 7 | 6353, 2767, 5865, 142, 1316 |
| 7393 | 5 | 4197, 2141, 1150, 7111, 5768 | 7417 | 5 | 5439, 1678, 7039, 209, 1490 |
| 7477 | 2 | 2950, 2906, 3345, 4985, 1 | 7489 | 7 | 2217, 568, 5351, 5449, 1520 |
| 7537 | 7 | 1574, 697, 2843, 1768, 6267 | 7549 | 2 | 1155, 1786, 1208, 1097, 1 |
| 7561 | 13 | 4133, 5050, 3423, 4837, 458 | 7573 | 2 | 1163, 3585, 1687, 1582, 2 |
| 7621 | 2 | 1949, 5398, 7496, 4797, 1 | 7669 | 2 | 201, 4595, 1468, 5497, 2 |
| 7681 | 17 | 3145, 2602, 483, 35, 7448 | 7717 | 2 | 952, 2283, 2371, 2249, 2 |
| 7741 | 7 | 640, 56, 2335, 1931, 3243 | 7753 | 10 | 1498, 6669, 857, 3764, 1921 |
| 7789 | 2 | 7508, 4725, 5147, 5314, 1 | 7873 | 5 | 4190, 1522, 3947, 7137, 1 |
| 7921 | * | 2818, 5487, 2900, 6245, 1 | 7933 | 2 | 7157, 4688, 1527, 1594, 1 |
| 7993 | 5 | 1037, 4609, 4413, 3514, 6080 | 8017 | 5 | 3929, 6883, 6268, 5259, 7496 |
| 8053 | 2 | 4083, 3923, 3094, 4699, 2 | 8089 | 17 | 6832, 641, 6805, 2451, 4958 |
| 8101 | 6 | 5864, 3970, 2113, 989, 4131 | 8161 | 7 | 6940, 1238, 7331, 6435, 1 |
| 8209 | 7 | 7601, 1, 411, 7948, 4124 | 8221 | 2 | 844, 1365, 4883, 4567, 2 |
| 8233 | 10 | 2086, 7133, 6763, 3567, 6032 | 8269 | 2 | 1013, 3830, 2098, 7623, 1 |
| 8293 | 2 | 8051, 3, 6770, 1426, 1 | 8317 | 6 | 5308, 1, 4376, 3227, 7911 |
| 8329 | 7 | 7973, 1522, 273, 6289, 2894 | 8353 | 5 | 4653, 4078, 5024, 4307, 1 |
| 8377 | 5 | 171, 5057, 7150, 7250, 199 | 8389 | 6 | 4739, 1, 5434, 2612, 5235 |
| 8461 | 6 | 7957, 7955, 2049, 4990, 1490 | 8521 | 13 | 5831, 5323, 3201, 3166, 5282 |
| 8581 | 6 | 3208, 1, 7751, 7569, 7358 | 8629 | 6 | 1493, 3248, 1510, 2329, 3483 |
| 8641 | 17 | 2323, 6071, 7204, 6219, 2588 | 8677 | 2 | 4865, 598, 5421, 4273, 2 |
| 8689 | 13 | 2547, 1129, 394, 7991, 7484 | 8713 | 5 | 7883, 5132, 6970, 2163, 1 |
| 8737 | 5 | 8620, 1760, 4097, 3693, 1 | 8761 | 23 | 4215, 3484, 3919, 8735, 7568 |
| 8821 | 2 | 2853, 2558, 1552, 7193, 1 | 8893 | 5 | 3374, 5637, 2284, 2627, 1 |
| 8929 | 11 | 1137, 4030, 5549, 7136, 1 | 8941 | 6 | 3616, 1733, 5888, 1633, 8007 |
| 9001 | 7 | 4805, 6578, 4696, 2397, 1 | 9013 | 5 | 1997, 5744, 6931, 7024, 2007 |
| 9049 | 7 | 2632, 8579, 2875, 6219, 476 | 9109 | 10 | 2881, 1294, 8126, 7865, 6165 |
| 9133 | 6 | 3463, 7312, 3665, 4268, 1821 | 9157 | 6 | 7774, 3039, 2551, 1367, 3038 |
| 9181 | 2 | 5593, 6562, 5247, 215, 2 | 9241 | 13 | 3199, 3448, 6035, 3801, 1724 |
| 9277 | 5 | 2927, 5668, 5318, 213, 1 | 9337 | 5 | 4231, 8061, 2638, 1379, 3788 |
| 9349 | 2 | 3465, 2, 4108, 6617, 1 | 9397 | 2 | 722, 5645, 1780, 8481, 1 |
| 9409 | * | 5535, 3701, 2044, 2275, 3332 | 9421 | 2 | 2479, 2542, 5015, 783, 2 |
| 9433 | 5 | 4185, 1930, 2329, 1865, 9386 | 9601 | 13 | 4179, 8104, 8735, 7339, 4052 |
| 9613 | 2 | 262, 765, 6659, 9601, 2 | 9649 | 7 | 4675, 7546, 909, 5915, 5240 |
| 9661 | 2 | 8187, 3110, 8069, 9430, 1 | 9697 | 10 | 3139, 6771, 8663, 3316, 5816 |
| 9721 | 7 | 3465, 563, 6880, 6110, 1 | 9733 | 2 | 6674, 5, 8038, 2763, 1 |
| 9769 | 13 | 2777, 5194, 3571, 9483, 3200 | 9781 | 6 | 1718, 5926, 4673, 2059, 8475 |
| 9817 | 5 | 9469, 2, 9148, 4577, 4443 | 9829 | 10 | 4370, 4546, 6687, 5969, 8407 |
| 9901 | 2 | 1834, 9133, 1059, 9683, 2 | 9949 | 2 | 263, 1419, 3937, 9178, 2 |
| 9973 | 11 | 6520, 4903, 9071, 3662, 3093 | 10009 | 11 | 6817, 6207, 8824, 2273, 7316 |
| 10069 | 2 | 2056, 2067, 6362, 6713, 1 | 10093 | 2 | 6190, 6584, 7863, 2381, 1 |
| 10141 | 2 | 3969, 5740, 1232, 7697, 1 | 10177 | 7 | 9400, 7124, 4405, 2315, 3993 |
| 10201 | * | 9405, 3899, 1160, 9808, 1 | 10273 | 10 | 8907, 346, 2515, 5711, 7220 |
| 10321 | 7 | 1981, 1480, 1239, 3815, 7754 | 10333 | 5 | 2227, 3284, 10294, 1853, 3069 |
| 10357 | 2 | 1279, 4923, 2231, 7870, 2 | 10369 | 13 | 5961, 2716, 995, 6163, 6818 |
| 10429 | 7 | 2225, 3010, 6566, 1411, 7095 | 10453 | 5 | 10052, 9964, 7733, 303, 325 |

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD $13^q$ construction (cont.)

| $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ | $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ |
|---|---|---|---|---|---|
| 10477 | 2 | 5572, 2347, 2703, 6329, 2 | 10501 | 2 | 5092, 7214, 1889, 3045, 1 |
| 10513 | 7 | 667, 10300, 8949, 17, 6284 | 10597 | 5 | 7725, 2794, 5741, 4232, 1 |
| 10609 | * | 7241, 208, 3735, 9541, 9152 | 10657 | 7 | 3892, 1, 716, 9857, 9501 |
| 10729 | 7 | 2597, 5218, 1735, 8523, 10004 | 10753 | 11 | 4370, 7025, 8671, 9082, 5499 |
| 10789 | 2 | 3621, 9370, 3764, 5813, 1 | 10837 | 2 | 10499, 10256, 8416, 9819, 1 |
| 10861 | 2 | 7874, 3802, 6413, 2685, 1 | 10909 | 2 | 6975, 6086, 953, 4708, 1 |
| 10957 | 5 | 5657, 4732, 8073, 4682, 1 | 10993 | 7 | 5767, 10335, 9046, 5633, 8420 |
| 11113 | 13 | 1673, 10982, 6982, 3793, 393 | 11149 | 10 | 9781, 3316, 10973, 8450, 6813 |
| 11161 | 7 | 10901, 550, 4779, 4676, 1 | 11173 | 5 | 9737, 1629, 7558, 4730, 1 |
| 11197 | 2 | 3469, 10721, 7149, 5374, 2 | 11257 | 10 | 9147, 2776, 4067, 7147, 3272 |
| 11317 | 2 | 203, 10268, 9994, 1479, 1 | 11329 | 7 | 213, 6988, 7721, 10508, 1 |
| 11353 | 7 | 8629, 10726, 10593, 7595, 2834 | 11437 | 2 | 320, 2109, 3965, 6706, 1 |
| 11449 | * | 6412, 5726, 6639, 3527, 1 | 11497 | 7 | 7287, 11123, 8938, 49, 830 |
| 11593 | 5 | 11291, 11320, 11233, 7743, 9860 | 11617 | 10 | 5275, 5416, 10059, 9725, 6992 |
| 11677 | 2 | 7840, 4501, 3135, 10331, 2 | 11689 | 7 | 4859, 6154, 6565, 1671, 9446 |
| 11701 | 6 | 5817, 4330, 9055, 6545, 9536 | 11821 | 2 | 4139, 3, 1226, 8260, 2023 |
| 11833 | 5 | 7078, 10259, 11361, 9961, 8684 | 11881 | * | 10097, 1210, 4101, 9145, 8360 |
| 11941 | 10 | 2782, 2273, 10995, 11906, 5575 | 11953 | 5 | 8199, 11494, 3653, 1862, 1 |
| 12037 | 5 | 10313, 5084, 5901, 4726, 3043 | 12049 | 13 | 10717, 10048, 2733, 2099, 9290 |
| 12073 | 7 | 7642, 8768, 3403, 875, 3219 | 12097 | 5 | 4317, 50, 3581, 3406, 1 |
| 12109 | 6 | 11224, 1, 6845, 10160, 10953 | 12157 | 2 | 10697, 11924, 6730, 2199, 1 |
| 12241 | 7 | 8695, 4774, 131, 2259, 1994 | 12253 | 2 | 10239, 4888, 545, 4952, 1 |
| 12277 | 2 | 2477, 7462, 2420, 807, 1 | 12289 | 11 | 4579, 3472, 3231, 1529, 4172 |
| 12301 | 2 | 4167, 5932, 767, 6062, 1 | 12373 | 2 | 8433, 962, 12239, 6826, 1 |
| 12409 | 7 | 10269, 2488, 383, 7537, 7448 | 12421 | 7 | 9223, 3604, 7697, 1388, 10581 |
| 12433 | 13 | 10913, 1, 8362, 2697, 6362 | 12457 | 10 | 8753, 3400, 6709, 9, 6800 |
| 12517 | 6 | 11585, 2404, 4442, 10651, 10113 | 12541 | 14 | 4712, 6400, 11253, 863, 4735 |
| 12553 | 5 | 4555, 7852, 8763, 8177, 9350 | 12577 | 10 | 7849, 513, 340, 9017, 4880 |
| 12589 | 2 | 9069, 38, 11554, 12437, 1 | 12601 | 11 | 9855, 8836, 11513, 4081, 10718 |
| 12613 | 2 | 4162, 3, 8078, 10811, 1 | 12637 | 2 | 11750, 9256, 5535, 1871, 1 |
| 12697 | 7 | 4513, 8254, 12689, 8445, 2468 | 12721 | 13 | 11773, 6280, 7433, 9129, 1934 |
| 12757 | 2 | 1991, 4261, 10096, 8457, 2 | 12769 | * | 5753, 2631, 1798, 4526, 1 |
| 12781 | 2 | 6881, 3579, 1228, 1706, 1 | 12829 | 2 | 6239, 6944, 8733, 922, 1 |
| 12841 | 21 | 10096, 1, 11957, 7737, 9590 | 12853 | 5 | 9506, 11871, 9844, 6965, 1 |
| 12889 | 13 | 11357, 9982, 7963, 10251, 5864 | 12973 | 14 | 10349, 10996, 3031, 12866, 237 |
| 13009 | 7 | 9322, 12525, 617, 9343, 2582 | 13033 | 5 | 8825, 10012, 1927, 10983, 5006 |
| 13093 | 6 | 7240, 1, 3209, 11480, 5931 | 13177 | 5 | 8962, 8720, 603, 6737, 1 |
| 13249 | 7 | 4685, 11025, 7900, 7243, 11582 | 13297 | 5 | 2360, 11794, 6155, 4023, 12055 |
| 13309 | 6 | 10667, 640, 10381, 5288, 639 | 13381 | 10 | 658, 2330, 7595, 3849, 7855 |
| 13417 | 5 | 3769, 12574, 3533, 13101, 10640 | 13441 | 11 | 6493, 8032, 8651, 13383, 13328 |
| 13477 | 2 | 6733, 2578, 5673, 8891, 2 | 13513 | 5 | 863, 1240, 2612, 13161, 1 |
| 13537 | 7 | 6519, 5530, 7385, 13435, 9542 | 13597 | 5 | 8062, 2756, 7693, 8909, 723 |
| 13633 | 5 | 3893, 4660, 10101, 12098, 1 | 13669 | 6 | 8775, 4307, 10276, 6068, 1 |
| 13681 | 22 | 12161, 12352, 3097, 8378, 2769 | 13693 | 6 | 215, 6652, 829, 1238, 7041 |
| 13729 | 23 | 11331, 11812, 3691, 125, 4220 | 13789 | 7 | 6820, 489, 10753, 12431, 10700 |
| 13873 | 5 | 2207, 3838, 6159, 4279, 4076 | 13921 | 7 | 12993, 316, 12301, 13397, 632 |
| 13933 | 2 | 11753, 2, 5278, 5289, 1 | 14029 | 6 | 8572, 1532, 6635, 7717, 13263 |
| 14149 | 6 | 4133, 13268, 7540, 3019, 2205 | 14173 | 2 | 11009, 6339, 2068, 2587, 2 |
| 14197 | 11 | 2474, 6664, 9283, 12821, 8331 | 14221 | 2 | 7486, 3, 13994, 203, 7951 |
| 14281 | 19 | 1381, 7468, 9777, 725, 12674 | 14293 | 6 | 9688, 2936, 3287, 13261, 12825 |
| 14341 | 2 | 12058, 2411, 5929, 9285, 2 | 14389 | 2 | 5565, 3196, 11522, 2987, 1 |
| 14401 | 11 | 2269, 10264, 12207, 4373, 2966 | 14437 | 5 | 11198, 8989, 13529, 3268, 1689 |
| 14449 | 22 | 3793, 7912, 11661, 2945, 3956 | 14461 | 2 | 12538, 12488, 3605, 5193, 1 |
| 14533 | 2 | 227, 4123, 10305, 4036, 2 | 14557 | 2 | 8693, 8662, 949, 2541, 2 |
| 14593 | 5 | 8919, 4184, 832, 3431, 1 | 14629 | 2 | 12635, 9776, 11890, 13401, 13699 |
| 14641 | * | 13628, 8781, 12760, 2183, 1 | 14653 | 2 | 7517, 3092, 5182, 10527, 1 |
| 14713 | 5 | 14411, 13996, 759, 2923, 13280 | 14737 | 10 | 14483, 1588, 9177, 1291, 3176 |
| 14797 | 2 | 3988, 11867, 12038, 12813, 1 | 14821 | 2 | 9781, 7677, 9778, 8819, 2 |
| 14869 | 2 | 983, 1838, 4479, 10984, 1 | 14929 | 7 | 3860, 12568, 2105, 6633, 1 |
| 15013 | 2 | 6383, 4, 11762, 13167, 1 | 15061 | 2 | 5417, 12561, 14528, 11380, 1 |
| 15073 | 5 | 10107, 4097, 4504, 11870, 1 | 15121 | 11 | 10325, 952, 3471, 6211, 9380 |
| 15193 | 5 | 3275, 3362, 5055, 13234, 1 | 15217 | 10 | 14725, 12508, 10883, 13305, 3668 |
| 15241 | 11 | 13799, 7000, 8307, 3265, 11120 | 15277 | 6 | 2569, 14104, 8285, 12674, 1173 |
| 15289 | 11 | 8769, 2422, 5189, 12217, 338 | 15313 | 5 | 7054, 11456, 12633, 1769, 1 |
| 15349 | 2 | 7941, 12485, 1054, 1142, 1 | 15361 | 7 | 11158, 9029, 9025, 11631, 2696 |
| 15373 | 2 | 165, 851, 9062, 3406, 1 | 15493 | 5 | 9698, 682, 2173, 2567, 3033 |
| 15541 | 6 | 12461, 8080, 9865, 13107, 11810 | 15601 | 23 | 1142, 5374, 4081, 1109, 5055 |
| 15625 | * | 3820, 11714, 6323, 7011, 1 | 15649 | 11 | 8515, 14746, 14999, 1335, 2720 |
| 15661 | 2 | 3713, 6602, 5355, 1942, 1 | 15733 | 6 | 1526, 4348, 10855, 7241, 2955 |
| 15817 | 5 | 125, 1052, 5752, 6261, 1 | 15877 | 5 | 5047, 10826, 5920, 14615, 3975 |
| 15889 | 21 | 5039, 3550, 8229, 12499, 1046 | 15901 | 10 | 7481, 7934, 15880, 11017, 9111 |
| 15913 | 5 | 11459, 10305, 8302, 6050, 10399 | 15937 | 7 | 9250, 11168, 10333, 9701, 10641 |
| 15973 | 7 | 4856, 967, 826, 2945, 5979 | 16033 | 5 | 7495, 11842, 8957, 11703, 7652 |
| 16057 | 7 | 1601, 8284, 12109, 614, 663 | 16069 | 2 | 8763, 2, 658, 7853, 1 |
| 16129 | * | 875, 4864, 16089, 13435, 9344 | 16141 | 6 | 10300, 8495, 15501, 6697, 7646 |
| 16149 | 2 | 8405, 6140, 465, 15772, 1 | 16249 | 17 | 14037, 1552, 16165, 10781, 3452 |
| 16273 | 7 | 15827, 295, 3592, 933, 4628 | 16333 | 2 | 7953, 14950, 12290, 3509, 1 |
| 16369 | 7 | 1409, 1, 1534, 1053, 2384 | 16381 | 2 | 1078, 10208, 15975, 6767, 1 |
| 16417 | 10 | 14783, 11281, 16191, 6928, 12104 | 16453 | 2 | 1856, 8164, 6993, 14711, 1 |
| 16477 | 2 | 11056, 7153, 2219, 10755, 2 | 16561 | 7 | 15981, 14656, 14819, 5479, 16112 |
| 16573 | 2 | 8201, 4693, 2703, 1870, 2 | 16633 | 15 | 14043, 5080, 1337, 3529, 1640 |
| 16657 | 5 | 938, 3088, 213, 12755, 9163 | 16693 | 2 | 903, 4, 4823, 12667, 2 |
| 16729 | 13 | 15853, 7671, 13667, 5098, 4898 | 16741 | 6 | 16262, 8080, 2537, 5755, 8661 |
| 16921 | 17 | 903, 94, 3907, 983, 12920 | 16981 | 2 | 12410, 7629, 10691, 2914, 1 |
| 16993 | 10 | 8063, 1474, 6105, 11467, 2948 | 17029 | 10 | 4799, 11056, 4064, 2845, 14847 |
| 17041 | 7 | 4750, 1, 16395, 13481, 11060 | 17053 | 2 | 3249, 7177, 11771, 4270, 2 |
| 17077 | 2 | 5452, 6038, 12981, 14411, 1 | 17137 | 5 | 16363, 9904, 15381, 2165, 16760 |
| 17161 | * | 4906, 14552, 12299, 9219, 1 | 17209 | 14 | 12434, 12448, 15383, 6031, 11241 |
| 17257 | 5 | 4299, 6658, 5209, 953, 7484 | 17293 | 7 | 7525, 13666, 12992, 15287, 9489 |
| 17317 | 2 | 3069, 12844, 14375, 277, 2 | 17341 | 6 | 7412, 1, 1756, 3125, 5409 |
| 17377 | 7 | 9255, 8428, 14375, 9469, 16856 | 17389 | 2 | 1773, 2, 11843, 10816, 1 |
| 17401 | 11 | 1067, 1, 11745, 2446, 2228 | 17449 | 14 | 1947, 12856, 8755, 161, 11186 |
| 17497 | 5 | 2977, 7811, 4252, 8679, 13184 | 17509 | 2 | 11475, 10085, 15400, 8480, 1 |
| 17569 | 11 | 547, 4708, 2259, 13199, 17564 | 17581 | 10 | 4835, 7910, 2661, 15742, 3955 |
| 17713 | 7 | 1135, 3784, 644, 6551, 7695 | 17737 | 7 | 4963, 4792, 9893, 7617, 9584 |
| 17749 | 2 | 10930, 4799, 2727, 8977, 2 | 17761 | 19 | 15973, 13648, 2741, 10383, 17426 |
| 17881 | 7 | 775, 2056, 11807, 10059, 10520 | 17929 | 11 | 2279, 5380, 17263, 1335, 16508 |
| 17977 | 5 | 7107, 5576, 2476, 6383, 1 | 17989 | 2 | 5878, 2120, 15059, 16155, 1 |
| 18013 | 2 | 976, 7430, 8669, 3, 1 | 18049 | 13 | 8999, 13900, 7587, 7273, 10544 |
| 18061 | 6 | 6355, 2146, 16703, 15686, 15915 | 18097 | 5 | 3813, 10132, 13933, 17651, 14114 |
| 18121 | 23 | 6781, 4336, 14120, 1337, 3633 | 18133 | 5 | 874, 10671, 8465, 2948, 1 |
| 18169 | 11 | 16143, 6280, 7169, 14239, 3140 | 18181 | 2 | 16186, 14168, 10931, 6819, 1 |
| 18217 | 7 | 721, 14410, 7593, 8369, 10604 | 18229 | 2 | 16997, 8533, 5728, 12675, 2 |
| 18253 | 5 | 3476, 14860, 3359, 5083, 10185 | 18289 | 13 | 2356, 16841, 15853, 13479, 14810 |
| 18301 | 6 | 7141, 1462, 14591, 10166, 16839 | 18313 | 10 | 4961, 17488, 17835, 3763, 15884 |
| 18397 | 6 | 12139, 3568, 16838, 3503, 14829 | 18433 | 5 | 13647, 6844, 9065, 11245, 6296 |
| 18457 | 5 | 6388, 4787, 11953, 3279, 9572 | 18481 | 13 | 8711, 8704, 11371, 10383, 3824 |
| 18493 | 2 | 13217, 3241, 9441, 17614, 2 | 18517 | 6 | 17864, 16918, 17189, 1909, 1599 |
| 18541 | 6 | 11845, 10774, 2249, 5228, 7767 | 18553 | 5 | 14653, 10852, 10793, 1425, 15734 |
| 18637 | 2 | 14335, 9087, 18418, 17093, 2 | 18661 | 10 | 3557, 2140, 2510, 9661, 6777 |
| 18757 | 2 | 2783, 1792, 10100, 6633, 1 | 18769 | * | 2691, 9521, 18226, 6482, 1 |
| 18793 | 5 | 6369, 358, 14327, 92, 1 | 18913 | 7 | 7823, 9352, 1963, 14157, 9290 |
| 18973 | 2 | 9700, 380, 14237, 2937, 1 | 19009 | 23 | 2763, 11044, 11191, 5981, 1190 |

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD $13^q$ construction (cont.)

| $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ | $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ |
|---|---|---|---|---|---|
| 19069 | 2 | 13649, 13610, 2746, 17367, 1 | 19081 | 17 | 17378, 10906, 4331, 18937, 10737 |
| 19141 | 2 | 10484, 1913, 4612, 18117, 1 | 19213 | 5 | 3860, 1, 12340, 1841, 10821 |
| 19237 | 2 | 6009, 17590, 2858, 2675, 1 | 19249 | 7 | 12475, 5728, 17927, 16701, 5222 |
| 19273 | 5 | 5090, 2404, 18543, 13925, 18127 | 19309 | 6 | 13244, 12454, 3859, 7475, 15423 |
| 19321 | * | 8091, 5740, 6007, 17573, 140 | 19333 | 2 | 15957, 13490, 12514, 13547, 1 |
| 19381 | 7 | 19346, 5315, 19054, 12451, 5463 | 19417 | 5 | 16322, 4281, 10588, 11411, 1 |
| 19429 | 6 | 18817, 11350, 3272, 15335, 8079 | 19441 | 13 | 12489, 1, 784, 15377, 11894 |
| 19477 | 6 | 14753, 12616, 3817, 16850, 6861 | 19489 | 19 | 12171, 7666, 6565, 14957, 14276 |
| 19501 | 2 | 9488, 10853, 3508, 18699, 1 | 19597 | 2 | 12695, 3586, 6776, 16701, 1 |
| 19609 | 13 | 11593, 13265, 6591, 8716, 3680 | 19681 | 11 | 10547, 7420, 2911, 17169, 17444 |
| 19717 | 2 | 1961, 2, 5871, 2266, 1 | 19753 | 5 | 16875, 11984, 5483, 12040, 1 |
| 19777 | 11 | 15206, 1, 5944, 17477, 8325 | 19801 | 13 | 9463, 19775, 15298, 1509, 15560 |
| 19813 | 2 | 5219, 67, 15555, 8620, 2 | 19861 | 11 | 5533, 8045, 3548, 13768, 5073 |
| 19993 | 10 | 11195, 9028, 6367, 13971, 18056 | 20029 | 2 | 19931, 18508, 12998, 1059, 1 |
| 20089 | 7 | 10737, 17386, 18475, 9197, 9188 | 20101 | 6 | 14444, 3382, 14987, 4069, 16719 |
| 20113 | 10 | 11595, 11512, 5231, 14893, 2912 | 20149 | 2 | 10792, 17043, 8918, 19313, 2209 |
| 20161 | 13 | 16571, 18412, 8161, 16713, 19286 | 20173 | 2 | 6226, 8593, 5195, 19797, 2 |
| 20233 | 5 | 6205, 2133, 7331, 5794, 956 | 20269 | 2 | 6671, 6487, 14559, 6586, 2 |
| 20341 | 2 | 18440, 13773, 9533, 19468, 1 | 20353 | 5 | 9981, 18646, 17479, 14681, 15032 |
| 20389 | 6 | 10556, 13589, 7912, 2197, 14961 | 20509 | 2 | 16353, 2, 2272, 16691, 1 |
| 20521 | 11 | 791, 1582, 7721, 9655, 9176 | 20533 | 2 | 8788, 835, 15423, 3677, 2 |
| 20593 | 5 | 18965, 8690, 19372, 19797, 13255 | 20641 | 7 | 1533, 1, 3977, 10546, 11768 |
| 20749 | 2 | 15429, 4, 12659, 14161, 2 | 20773 | 2 | 5219, 17143, 4719, 14920, 2 |
| 20809 | 7 | 895, 8242, 339, 7949, 9512 | 20857 | 10 | 11091, 19276, 10901, 1561, 17696 |
| 20929 | 7 | 14727, 17182, 17905, 683, 12710 | 21001 | 11 | 1945, 15670, 13751, 5631, 8192 |
| 21013 | 2 | 19089, 19526, 20782, 8021, 1 | 21061 | 7 | 16316, 15310, 13991, 9589, 18045 |
| 21121 | 19 | 19571, 3412, 6211, 11907, 1706 | 21157 | 2 | 15061, 6135, 11818, 2837, 2 |
| 21169 | 13 | 11236, 1, 6791, 16365, 18290 | 21193 | 11 | 17785, 20638, 9107, 392, 15081 |
| 21277 | 6 | 1490, 7174, 1577, 8503, 14103 | 21313 | 5 | 13545, 11299, 19138, 14795, 10700 |
| 21397 | 2 | 21159, 985, 1847, 15532, 2 | 21433 | 5 | 21021, 10544, 11110, 5759, 1 |
| 21481 | 13 | 9911, 9742, 7923, 17846, 1 | 21493 | 2 | 15766, 5720, 10683, 4355, 1 |
| 21517 | 5 | 9233, 5829, 12112, 10406, 1 | 21529 | 11 | 12809, 10156, 9633, 10915, 1886 |
| 21577 | 5 | 298, 19790, 19593, 8915, 18757 | 21589 | 2 | 829, 21255, 7756, 15785, 2 |
| 21601 | 7 | 19205, 4060, 8383, 8553, 8120 | 21613 | 2 | 8770, 15104, 10883, 11235, 1 |
| 21649 | 14 | 13345, 19786, 15147, 2591, 17924 | 21661 | 2 | 13289, 2, 13576, 6999, 1 |
| 21673 | 10 | 3987, 19114, 809, 21313, 14456 | 21757 | 5 | 12089, 5368, 20198, 3057, 14581 |
| 21817 | 7 | 557, 20132, 7105, 17578, 16629 | 21841 | 11 | 18833, 20860, 8331, 5809, 19880 |
| 21937 | 7 | 17663, 14936, 907, 16342, 8325 | 21961 | 17 | 6569, 11968, 10807, 7827, 18182 |
| 21997 | 7 | 13232, 3982, 3499, 6551, 11559 | 22093 | 6 | 7373, 13552, 6302, 15181, 8541 |
| 22129 | 19 | 7777, 18921, 6004, 2627, 4874 | 22153 | 5 | 4061, 3850, 12549, 7093, 15458 |
| 22189 | 2 | 21467, 7724, 1672, 8487, 1 | 22201 | * | 14560, 11249, 20114, 20571, 1 |
| 22273 | 5 | 7805, 10099, 4138, 22029, 596 | 22369 | 11 | 5089, 1366, 18023, 1239, 17672 |
| 22381 | 10 | 5569, 11956, 11063, 873, 6356 | 22441 | 14 | 10687, 18459, 4907, 7204, 1718 |
| 22453 | 5 | 18431, 5140, 21008, 22225, 4383 | 22501 | 2 | 8529, 13462, 3722, 7727, 1 |
| 22549 | 2 | 4331, 2, 12666, 14253, 1 | 22573 | 6 | 22438, 11153, 13563, 6103, 11420 |
| 22621 | 2 | 22301, 4522, 5852, 16743, 1 | 22669 | 2 | 20743, 11458, 11573, 1875, 2 |
| 22717 | 2 | 16601, 1621, 3549, 5374, 2 | 22741 | 7 | 8266, 16178, 3989, 943, 22137 |
| 22777 | 7 | 19168, 1393, 15017, 12014, 6021 | 22801 | * | 4099, 11704, 3467, 5409, 6080 |
| 22861 | 2 | 3814, 21869, 4903, 747, 2 | 22921 | 7 | 2673, 18076, 8381, 7267, 17522 |
| 22993 | 5 | 2377, 18682, 8511, 12539, 17906 | 23017 | 5 | 5391, 21578, 16444, 6623, 1 |
| 23029 | 2 | 8474, 243, 19613, 16468, 1 | 23041 | 11 | 15395, 9202, 19899, 703, 19196 |
| 23053 | 2 | 17314, 15769, 5189, 7671, 2 | 23173 | 5 | 8495, 14871, 20900, 22918, 1 |
| 23197 | 2 | 7847, 489, 21439, 10162, 2 | 23209 | 31 | 1400, 21868, 19367, 7681, 4629 |
| 23269 | 6 | 22451, 1018, 21259, 7682, 22251 | 23293 | 5 | 7549, 22196, 19234, 3035, 5295 |
| 23473 | 5 | 8302, 8873, 14384, 10071, 1069 | 23497 | 5 | 153, 1162, 16109, 11719, 5216 |
| 23509 | 2 | 5279, 3712, 18074, 4857, 1 | 23557 | 5 | 4846, 1220, 19029, 16667, 7423 |
| 23581 | 6 | 21805, 22840, 13424, 1727, 741 | 23593 | 5 | 14337, 3658, 719, 22723, 7316 |
| 23629 | 2 | 20212, 19670, 737, 7089, 1 | 23677 | 5 | 4415, 9142, 12349, 7310, 4185 |
| 23689 | 11 | 22135, 11572, 18041, 18513, 5786 | 23761 | 7 | 9117, 17620, 18847, 9939, 10520 |
| 23773 | 5 | 23465, 21261, 23738, 17752, 1 | 23833 | 5 | 5848, 2325, 3155, 5042, 10393 |
| 23857 | 5 | 15519, 18464, 22048, 11903, 1 | 23869 | 2 | 16528, 7595, 17834, 15105, 2707 |
| 23893 | 5 | 23223, 19268, 11554, 5735, 6061 | 23917 | 2 | 20105, 2, 17434, 11727, 1 |
| 23929 | 7 | 23177, 12862, 4137, 9913, 7520 | 23977 | 5 | 22695, 4822, 1007, 10226, 1 |
| 24001 | 14 | 10693, 7492, 2913, 12335, 7892 | 24049 | 19 | 10033, 5878, 2757, 5993, 5984 |
| 24061 | 10 | 2239, 7622, 2944, 12593, 2283 | 24097 | 5 | 1489, 23188, 14493, 11825, 3206 |
| 24109 | 2 | 15250, 19773, 8923, 19367, 2 | 24121 | 13 | 9292, 19287, 4825, 9347, 6038 |
| 24133 | 6 | 9854, 2344, 15437, 3601, 21789 | 24169 | 11 | 17881, 2710, 10889, 9218, 19485 |
| 24181 | 17 | 4159, 17026, 17135, 18608, 5319 | 24229 | 2 | 154, 977, 19070, 4599, 1 |
| 24337 | 5 | 20690, 21209, 8493, 16090, 12877 | 24373 | 7 | 15205, 6071, 6506, 23170, 23643 |
| 24421 | 7 | 10826, 20561, 5932, 14617, 22041 | 24469 | 14 | 9991, 22360, 8366, 155, 4557 |
| 24481 | 11 | 2843, 9748, 20497, 10821, 15842 | 24517 | 5 | 7081, 21448, 10655, 8882, 22425 |
| 24649 | * | 10957, 2212, 14399, 10251, 1106 | 24697 | 5 | 2223, 7667, 727, 21388, 15332 |
| 24709 | 2 | 2445, 2, 526, 19565, 1 | 24733 | 2 | 1594, 12193, 2085, 12785, 2 |
| 24781 | 2 | 22864, 3, 10940, 4415, 1 | 24793 | 5 | 16743, 4088, 155, 4936, 1 |
| 24841 | 14 | 1671, 20692, 3499, 15983, 16544 | 24877 | 5 | 2182, 10347, 9761, 7790, 1 |
| 24889 | 11 | 7233, 10822, 23467, 11567, 11552 | 25033 | 5 | 11457, 24346, 23191, 24659, 11612 |
| 25057 | 5 | 829, 18556, 5487, 8735, 12056 | 25117 | 5 | 1145, 21746, 8686, 3801, 2899 |
| 25153 | 10 | 145, 24922, 23259, 23987, 15062 | 25189 | 2 | 9609, 23740, 18320, 10265, 1 |
| 25237 | 2 | 14853, 2848, 22229, 14779, 2 | 25261 | 7 | 21826, 9092, 4553, 14551, 16671 |
| 25309 | 13 | 21256, 20054, 12767, 11235, 1369 | 25321 | 19 | 1169, 14548, 15061, 7245, 19334 |
| 25357 | 2 | 23795, 18664, 20474, 4911, 1 | 25453 | 2 | 9491, 24296, 11914, 2349, 1 |
| 25537 | 10 | 17443, 11104, 4061, 10071, 18320 | 25561 | 11 | 7351, 5992, 6975, 12671, 2996 |
| 25609 | 7 | 13495, 15892, 8255, 19041, 20750 | 25621 | 10 | 14599, 6478, 5435, 23924, 4815 |
| 25633 | 5 | 7259, 15532, 19773, 5605, 13544 | 25657 | 5 | 11699, 21400, 6524, 5781, 1 |
| 25693 | 2 | 4173, 23828, 18058, 7319, 1 | 25717 | 2 | 6893, 4154, 9297, 8446, 1 |
| 25741 | 6 | 23192, 17596, 8551, 11111, 8145 | 25801 | 7 | 22601, 1, 10264, 2661, 16388 |
| 25849 | 7 | 12343, 25660, 3303, 21233, 19568 | 25873 | 10 | 5439, 13210, 24911, 18751, 548 |
| 25933 | 2 | 20561, 2, 16036, 12867, 1 | 25969 | 7 | 25829, 22348, 16993, 1935, 24158 |
| 25981 | 11 | 5972, 1, 16408, 11861, 1203 | 26017 | 5 | 15062, 5499, 20975, 12244, 1 |
| 26029 | 6 | 25939, 15934, 1832, 13961, 10095 | 26041 | 13 | 25093, 21586, 21029, 23367, 6356 |
| 26053 | 2 | 23097, 4256, 11903, 13126, 1 | 26113 | 7 | 6791, 1, 9016, 18507, 404 |
| 26161 | 13 | 8559, 1, 24298, 3785, 20210 | 26209 | 11 | 9081, 25360, 21437, 20815, 23402 |
| 26293 | 6 | 13061, 1546, 5869, 13670, 24747 | 26317 | 6 | 22331, 22060, 5078, 15565, 4257 |
| 26437 | 5 | 16922, 13923, 4853, 14116, 1 | 26449 | 7 | 25171, 11926, 23687, 24861, 2024 |
| 26497 | 5 | 8991, 16004, 8518, 16523, 1 | 26557 | 2 | 1293, 7102, 18476, 16001, 1 |
| 26569 | * | 4245, 9184, 12937, 22949, 17876 | 26641 | 7 | 15245, 1, 16047, 6886, 8024 |
| 26701 | 22 | 3361, 15712, 16814, 4733, 14385 | 26713 | 10 | 12861, 18358, 18031, 21017, 11756 |
| 26737 | 10 | 19453, 22966, 8573, 3081, 19196 | 26821 | 2 | 21575, 25639, 20488, 16671, 2 |
| 26833 | 5 | 5338, 5486, 19187, 9531, 1 | 26881 | 11 | 9763, 17164, 20651, 10239, 12548 |
| 26893 | 5 | 25828, 1881, 671, 23360, 1 | 26953 | 7 | 14146, 5420, 2917, 15251, 6117 |
| 27061 | 2 | 2829, 2, 16396, 8465, 1 | 27073 | 5 | 18143, 8692, 17277, 16285, 11402 |
| 27109 | 7 | 18932, 982, 11243, 9589, 8577 | 27241 | 17 | 8681, 16186, 55, 22329, 16580 |
| 27253 | 2 | 17019, 22813, 665, 6724, 2 | 27277 | 6 | 6623, 2284, 15829, 19112, 24993 |
| 27337 | 5 | 22195, 12754, 23943, 20915, 25508 | 27361 | 7 | 12112, 1, 6167, 19299, 21146 |
| 27397 | 2 | 700, 10883, 23108, 27051, 1 | 27409 | 13 | 5555, 22234, 9945, 8101, 15722 |
| 27457 | 7 | 741, 12082, 668, 7277, 8671 | 27481 | 7 | 12287, 1390, 27445, 13287, 17672 |
| 27529 | 7 | 1457, 17620, 7237, 5253, 8462 | 27541 | 19 | 5741, 10959, 22603, 11308, 7064 |
| 27673 | 11 | 24494, 26026, 23657, 727, 9219 | 27697 | 5 | 10670, 9208, 12615, 10295, 1 |
| 27733 | 2 | 17259, 18860, 8110, 425, 1 | 27793 | 5 | 25881, 2662, 5483, 15817, 9224 |
| 27817 | 5 | 19604, 9784, 24065, 8289, 1 | 27889 | * | 14234, 27508, 83, 1209, 1 |
| 27901 | 2 | 17453, 2, 21130, 11511, 1 | 27961 | 13 | 22989, 25168, 8699, 17654, 21625 |
| 27997 | 5 | 27016, 16556, 14647, 14225, 4731 | 28057 | 5 | 1547, 16246, 13736, 17739, 1 |
| 28069 | 7 | 24575, 7209, 22471, 19060, 7490 | 28081 | 19 | 4749, 21202, 7891, 7097, 2258 |

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD $13^q$ construction (cont.)

| $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ | $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ |
|---|---|---|---|---|---|
| 28201 | 11 | 13367, 11596, 23349, 4357, 5342 | 28297 | 5 | 28211, 140, 2715, 13030, 1 |
| 28309 | 2 | 16095, 20812, 21086, 623, 1 | 28393 | 15 | 1262, 1540, 16699, 22511, 1509 |
| 28429 | 2 | 7839, 2, 27724, 10529, 1 | 28477 | 2 | 7227, 4, 21715, 24917, 2 |
| 28513 | 5 | 9717, 21700, 5665, 1889, 22136 | 28537 | 5 | 6430, 27338, 24225, 21203, 27301 |
| 28549 | 2 | 3335, 2, 892, 21501, 1 | 28561 | * | 22209, 8311, 8512, 13451, 11900 |
| 28573 | 2 | 11817, 28297, 13349, 28258, 2 | 28597 | 2 | 26326, 14131, 5481, 17825, 2 |
| 28621 | 13 | 14416, 17455, 2303, 22214, 4065 | 28657 | 5 | 8913, 5584, 13789, 5741, 27932 |
| 28669 | 6 | 4087, 22144, 15620, 845, 6525 | 28729 | 22 | 9007, 23044, 485, 7341, 11522 |
| 28753 | 10 | 12919, 21868, 12969, 14477, 24272 | 28789 | 7 | 4907, 23095, 17014, 19190, 16149 |
| 28813 | 2 | 12927, 13825, 24748, 7733, 2 | 28837 | 2 | 11195, 8854, 9452, 24783, 1 |
| 28909 | 2 | 2573, 3, 27338, 11728, 26773 | 28921 | 11 | 19989, 1, 23266, 20291, 4856 |
| 28933 | 2 | 24923, 26564, 4918, 12813, 1 | 29017 | 5 | 7617, 6880, 28430, 22415, 1 |
| 29077 | 2 | 19209, 11452, 6938, 24941, 1 | 29101 | 2 | 6430, 9671, 27337, 7119, 2 |
| 29137 | 5 | 21095, 4360, 15079, 27525, 2180 | 29173 | 2 | 21439, 22409, 26469, 15046, 2 |
| 29209 | 7 | 17615, 21484, 12177, 29047, 15704 | 29221 | 2 | 27195, 2, 23926, 857, 1 |
| 29269 | 6 | 20483, 9472, 12409, 20396, 18399 | 29389 | 2 | 2871, 2, 8668, 28229, 1 |
| 29401 | 13 | 25797, 25006, 1729, 29375, 27890 | 29437 | 2 | 12663, 9188, 15724, 22751, 1 |
| 29473 | 5 | 23759, 16383, 19246, 3793, 15812 | 29569 | 17 | 6777, 1, 3820, 28271, 27170 |
| 29581 | 10 | 10589, 8330, 22359, 8014, 18955 | 29629 | 7 | 10160, 16501, 190, 8663, 29385 |
| 29641 | 7 | 7449, 3436, 9199, 6869, 1718 | 29761 | 17 | 1349, 12880, 8629, 22449, 22244 |
| 29833 | 5 | 24821, 18964, 9687, 11215, 14360 | 29881 | 7 | 9751, 5170, 8475, 12425, 4586 |
| 29917 | 2 | 22791, 25840, 28208, 29255, 1 | 29929 | * | 25288, 15137, 11079, 17276, 1 |
| 29989 | 2 | 13541, 6793, 10072, 5289, 2 | 30013 | 2 | 10547, 3, 15572, 22498, 1 |
| 30097 | 10 | 11347, 10612, 12525, 965, 12164 | 30109 | 2 | 14104, 7946, 19805, 24459, 1 |
| 30133 | 5 | 16330, 27092, 21113, 9553, 15627 | 30169 | 7 | 4853, 4732, 6615, 11209, 17450 |
| 30181 | 2 | 21392, 3646, 22509, 28229, 5329 | 30241 | 11 | 1901, 20944, 8965, 18201, 10472 |
| 30253 | 2 | 9047, 5410, 10832, 16395, 1 | 30313 | 5 | 20819, 21214, 23061, 541, 10682 |
| 30469 | 2 | 23643, 7562, 19493, 24880, 1 | 30493 | 6 | 1192, 27863, 17349, 21541, 2630 |
| 30517 | 2 | 3689, 24430, 20612, 15633, 1 | 30529 | 13 | 6719, 11224, 9301, 30501, 5612 |
| 30553 | 5 | 9611, 27868, 10927, 4047, 13934 | 30577 | 5 | 20411, 4558, 17540, 18195, 1 |
| 30637 | 2 | 30603, 11954, 1186, 21761, 1 | 30649 | 7 | 13197, 11572, 7817, 29131, 25778 |
| 30661 | 2 | 4270, 17066, 9209, 29499, 1 | 30697 | 10 | 7021, 19762, 7797, 3047, 18512 |
| 30757 | 5 | 5993, 25480, 2480, 27763, 24123 | 30781 | 2 | 6045, 15046, 16910, 10109, 1 |
| 30817 | 5 | 6963, 28456, 10261, 21785, 14228 | 30829 | 2 | 22779, 4270, 7466, 8507, 1 |
| 30841 | 7 | 13439, 14338, 4687, 7815, 3944 | 30853 | 2 | 30791, 3490, 5594, 6843, 1 |
| 30937 | 15 | 12313, 20056, 4562, 3353, 29109 | 30949 | 10 | 30680, 27802, 18107, 3361, 9441 |
| 31033 | 10 | 3323, 20944, 2643, 30193, 1496 | 31069 | 2 | 6983, 2, 18261, 14350, 1 |
| 31081 | 13 | 4443, 22708, 13103, 21883, 15518 | 31153 | 10 | 24185, 17116, 24309, 19087, 3080 |
| 31177 | 7 | 21988, 10100, 13697, 13873, 15603 | 31189 | 13 | 22541, 3484, 3434, 13297, 20583 |
| 31237 | 6 | 28064, 16054, 17845, 18425, 15183 | 31249 | 23 | 26105, 3933, 28906, 26863, 20 |
| 31321 | 7 | 3167, 24628, 28611, 12949, 12314 | 31333 | 5 | 23639, 30956, 12220, 19825, 11637 |
| 31357 | 2 | 15443, 6032, 16455, 25198, 1 | 31393 | 5 | 15949, 6034, 1259, 28263, 15572 |
| 31477 | 6 | 26993, 6286, 28573, 26468, 25191 | 31489 | 7 | 25751, 8638, 26109, 19801, 17276 |
| 31513 | 7 | 25885, 31180, 3497, 8625, 30848 | 31573 | 5 | 9538, 29054, 8201, 6283, 22119 |
| 31657 | 5 | 13191, 8804, 707, 1978, 1 | 31729 | 7 | 897, 6796, 3763, 9119, 21548 |
| 31741 | 6 | 28021, 30268, 1763, 29588, 1473 | 31849 | 14 | 89, 30238, 3727, 1923, 2618 |
| 31873 | 11 | 28127, 5260, 23744, 11419, 18861 | 31957 | 2 | 10511, 30662, 30993, 3190, 1 |
| 31981 | 6 | 28342, 5469, 22009, 17267, 8270 | 32029 | 2 | 20933, 20672, 28581, 28660, 13717 |
| 32041 | * | 17327, 23373, 18572, 9946, 1 | 32077 | 2 | 26458, 31963, 1937, 3459, 2 |
| 32089 | 13 | 23181, 18292, 9863, 22795, 344 | 32173 | 5 | 32029, 11794, 18128, 23507, 28653 |
| 32233 | 5 | 3971, 25413, 21286, 27373, 21944 | 32257 | 15 | 21259, 15754, 28755, 31871, 20084 |
| 32341 | 2 | 7689, 21517, 16528, 23087, 2 | 32353 | 15 | 17032, 31364, 4663, 29327, 495 |
| 32377 | 5 | 987, 23134, 21325, 20045, 14768 | 32401 | 7 | 28403, 10090, 20091, 16699, 20180 |
| 32413 | 5 | 23918, 15232, 24649, 28229, 21225 | 32497 | 7 | 9535, 17752, 10493, 711, 3008 |
| 32533 | 2 | 5507, 25016, 11134, 7647, 1 | 32569 | 7 | 1077, 16354, 21223, 25919, 19490 |
| 32653 | 2 | 18413, 6118, 12002, 6279, 1 | 32713 | 5 | 8342, 16943, 10192, 18639, 18577 |
| 32749 | 2 | 19559, 2, 5896, 30267, 1 | 32761 | * | 13381, 8008, 21345, 10325, 20384 |

Table 11: Table of $m, w, c, c', c''$ for BIBD$(q, 12, 1)$ construction ($q$ prime)

| $q$ | $m$ | $w$ | c | c' | c" | $q$ | $m$ | $w$ | c | c' | c" |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5413 | 2117 | 1224 | 3380 | 1941 | 4205 | 6073 | 2550 | 1842 | 5141 | 4875 | 1950 |
| 6337 | 1735 | 4824 | 4184 | 4073 | 3621 | 6469 | 2392 | 4992 | 64 | 6212 | 5069 |
| 6733 | 6378 | 6113 | 3475 | 4630 | 4821 | 6997 | 2492 | 4088 | 2164 | 472 | 615 |
| 7129 | 2795 | 5879 | 2230 | 5669 | 11 | 7393 | 5394 | 1717 | 4266 | 6499 | 3661 |
| 7789 | 3822 | 233 | 6811 | 131 | 6852 | 8053 | 6744 | 3497 | 7598 | 3995 | 3883 |
| 8317 | 7077 | 1286 | 2936 | 7191 | 4486 | 8581 | 6263 | 424 | 398 | 318 | 3532 |
| 8713 | 5904 | 6175 | 1471 | 4408 | 4881 | 9109 | 5770 | 3120 | 5282 | 458 | 1019 |
| 9241 | 7962 | 9074 | 8415 | 5595 | 1745 | 9769 | 4461 | 9507 | 1557 | 324 | 1054 |
| 9901 | 6181 | 99 | 64 | 2204 | 6095 | 10429 | 6208 | 2414 | 1751 | 8331 | 10309 |
| 10957 | 8942 | 3145 | 4014 | 7721 | 10310 | 11353 | 8040 | 2307 | 413 | 10442 | 2232 |
| 11617 | 10190 | 9023 | 7271 | 8981 | 5420 | 12277 | 7847 | 11877 | 8192 | 6044 | 10550 |
| 12409 | 4841 | 9597 | 9833 | 5210 | 3068 | 12541 | 11585 | 10157 | 11102 | 7160 | 7059 |
| 13597 | 12373 | 4429 | 5256 | 10396 | 2739 | 13729 | 5476 | 7019 | 10912 | 11558 | 12736 |
| 14389 | 7105 | 13956 | 16 | 12102 | 5801 | 14653 | 3546 | 3693 | 7641 | 9119 | 2268 |
| 15313 | 8635 | 6819 | 6148 | 8085 | 875 | 15973 | 12061 | 1034 | 4771 | 6347 | 5908 |
| 16369 | 13275 | 338 | 11785 | 9786 | 6172 | 16633 | 752 | 4010 | 2100 | 7929 | 14216 |
| 17029 | 8861 | 5519 | 2782 | 757 | 9953 | 17293 | 7309 | 17161 | 10772 | 11099 | 354 |
| 18217 | 17856 | 8397 | 343 | 8847 | 9126 | 18481 | 16454 | 14188 | 5522 | 7092 | 9654 |
| 19009 | 7663 | 1625 | 529 | 7438 | 4112 | 19141 | 2425 | 10002 | 128 | 7791 | 12756 |
| 19273 | 14875 | 17533 | 15625 | 17315 | 587 | 19801 | 4142 | 2184 | 9251 | 19613 | 19387 |
| 20593 | 13610 | 143 | 1520 | 14365 | 3129 | 20857 | 8688 | 2047 | 10 | 16830 | 1720 |
| 21121 | 11634 | 13952 | 1704 | 16841 | 15698 | 21517 | 4770 | 2190 | 18424 | 2063 | 6494 |
| 21649 | 5358 | 14609 | 6130 | 19529 | 16194 | 22441 | 559 | 20414 | 17196 | 5967 | 7782 |
| 22573 | 15853 | 688 | 1296 | 4175 | 14805 | 23497 | 14115 | 23231 | 10478 | 21327 | 10548 |
| 23629 | 11971 | 20558 | 2048 | 9757 | 15004 | 23761 | 16962 | 14985 | 16122 | 19830 | 8136 |
| 23893 | 2208 | 5681 | 10368 | 6725 | 23602 | 24421 | 15074 | 5562 | 19965 | 17054 | 3656 |
| 25609 | 22883 | 11961 | 22243 | 24951 | 5228 | 25741 | 15895 | 24210 | 12965 | 17830 | 22885 |
| 25873 | 21339 | 23045 | 855 | 1200 | 4209 | 27061 | 26910 | 26896 | 256 | 1861 | 20151 |
| 27457 | 16675 | 23868 | 27290 | 21883 | 23298 | 28513 | 463 | 12370 | 15625 | 290 | 3950 |
| 28909 | 2499 | 10917 | 512 | 25493 | 24225 | 29173 | 22565 | 9972 | 4 | 18046 | 8170 |
| 29437 | 14250 | 21777 | 8 | 24758 | 27785 | 29569 | 614 | 13058 | 289 | 3578 | 28764 |
| 29833 | 15989 | 22037 | 625 | 10215 | 8611 | 30097 | 15065 | 22771 | 9709 | 24623 | 6781 |
| 30493 | 9170 | 21936 | 15006 | 13583 | 2737 | 30757 | 10533 | 15146 | 16766 | 13653 | 20901 |
| 31153 | 26022 | 30976 | 3104 | 13202 | 12205 | 32077 | 24294 | 15305 | 512 | 22778 | 15884 |
| 32341 | 22315 | 17796 | 2 | 19646 | 6292 | | | | | | |

Table 12: Table of $m, w, c, c', c''$ for BIBD$(q, 12, 1)$ construction ($q$ composite)

| $q$ | $m$ | $w$ | c | c' | c" |
|---|---|---|---|---|---|
| 7921 | $14x + 15$ | $2x + 45$ | $50x + 3$ | $67x + 83$ | $19x + 51$ |
| 11881 | $89x + 13$ | 63 | $29x + 100$ | $92x + 69$ | $76x + 41$ |
| 17161 | $37x + 2$ | $51x + 25$ | $130x + 117$ | $93x + 43$ | $30x + 30$ |

Table 13: Table of $\gamma_1, \ldots, \gamma_5$ for RBIBD$(11q+1, 12, 1)$ construction

| $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ | $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ |
|---|---|---|---|---|---|
| 349 | 2 | 325, 34, 182, 341, 117 | 373 | 2 | 343, 364, 164, 105, 245 |
| 409 | 21 | 37, 118, 81, 266, 179 | 421 | 2 | 109, 334, 56, 401, 183 |
| 433 | 5 | 73, 243, 382, 155, 8 | 457 | 13 | 241, 416, 215, 376, 81 |
| 529 | * | 1, 239, 231, 284, 94 | 541 | 2 | 295, 503, 520, 315, 110 |
| 577 | 5 | 1, 353, 160, 129, 272 | 601 | 7 | 433, 75, 128, 23, 478 |
| 613 | 2 | 481, 375, 574, 149, 86 | 625 | * | 55, 494, 244, 399, 203 |
| 661 | 2 | 25, 519, 178, 191, 344 | 673 | 5 | 403, 641, 129, 578, 358 |
| 709 | 2 | 1, 47, 254, 154, 219 | 733 | 6 | 1, 267, 500, 587, 466 |
| 757 | 2 | 283, 341, 615, 152, 424 | 769 | 11 | 1, 560, 712, 525, 731 |
| 829 | 2 | 685, 8, 580, 449, 537 | 841 | * | 1, 423, 593, 86, 88 |
| 853 | 2 | 97, 112, 176, 99, 779 | 877 | 2 | 787, 700, 363, 773, 104 |
| 937 | 5 | 1, 111, 370, 305, 296 | 961 | * | 1, 640, 245, 525, 278 |
| 997 | 7 | 823, 34, 449, 207, 302 | 1009 | 11 | 103, 230, 803, 279, 610 |
| 1021 | 10 | 349, 717, 701, 1018, 506 | 1033 | 5 | 1, 166, 993, 401, 434 |
| 1069 | 6 | 1, 489, 836, 215, 616 | 1093 | 5 | 1, 459, 412, 284, 965 |
| 1117 | 2 | 1105, 155, 382, 315, 578 | 1129 | 11 | 1, 1100, 897, 934, 677 |
| 1153 | 5 | 163, 860, 531, 412, 947 | 1201 | 11 | 577, 977, 1048, 621, 200 |
| 1213 | 2 | 1, 220, 230, 105, 467 | 1237 | 2 | 1, 47, 110, 40, 1143 |
| 1249 | 7 | 1, 358, 1094, 527, 279 | 1297 | 10 | 1, 908, 1037, 10, 495 |
| 1321 | 13 | 565, 772, 831, 482, 1247 | 1369 | * | 1, 38, 681, 713, 292 |
| 1381 | 2 | 1, 424, 899, 1202, 1179 | 1429 | 6 | 1, 953, 21, 1180, 476 |
| 1453 | 2 | 1, 1409, 243, 106, 566 | 1489 | 14 | 1, 1336, 681, 1172, 731 |
| 1549 | 2 | 799, 886, 446, 539, 279 | 1597 | 11 | 1375, 976, 755, 1550, 1395 |
| 1609 | 7 | 1, 1096, 812, 1391, 879 | 1621 | 2 | 1549, 713, 1228, 9, 1046 |
| 1657 | 11 | 1, 1156, 1587, 818, 1451 | 1669 | 2 | 901, 569, 1292, 1342, 1083 |
| 1681 | * | 235, 933, 608, 1505, 652 | 1693 | 2 | 1, 206, 5, 724, 1413 |
| 1741 | 2 | 1, 1353, 358, 932, 779 | 1753 | 7 | 1, 1180, 1124, 1685, 1011 |
| 1777 | 5 | 1, 730, 620, 1157, 1047 | 1789 | 6 | 1, 1598, 1036, 125, 81 |
| 1801 | 11 | 1, 251, 273, 250, 1478 | 1849 | * | 1285, 1587, 346, 1193, 584 |
| 1861 | 2 | 547, 4, 1430, 131, 1125 | 1873 | 10 | 1, 527, 981, 170, 1222 |
| 1933 | 5 | 1, 1354, 1755, 743, 998 | 1993 | 5 | 37, 214, 914, 1259, 591 |
| 2017 | 5 | 1, 1131, 800, 1673, 316 | 2029 | 2 | 1981, 1899, 976, 644, 275 |
| 2053 | 2 | 1, 562, 1934, 443, 963 | 2089 | 7 | 1, 1804, 99, 101, 524 |
| 2113 | 5 | 1, 863, 15, 622, 1088 | 2137 | 10 | 1279, 44, 1535, 1732, 1227 |
| 2161 | 23 | 1, 273, 839, 2012, 574 | 2197 | * | 1, 220, 305, 93, 980 |
| 2209 | * | 211, 94, 1415, 476, 381 | 2221 | 2 | 529, 1709, 1204, 1280, 1791 |
| 2269 | 2 | 739, 1018, 755, 1917, 1736 | 2281 | 7 | 1, 1742, 1779, 1126, 1823 |
| 2293 | 2 | 1, 1217, 422, 1773, 2242 | 2341 | 7 | 1129, 1409, 1024, 591, 1982 |
| 2377 | 5 | 511, 831, 1786, 236, 917 | 2389 | 2 | 2347, 662, 2169, 1864, 95 |
| 2401 | * | 1, 2339, 796, 1473, 1730 | 2437 | 2 | 1, 1114, 2403, 1535, 2138 |
| 2473 | 5 | 1, 657, 238, 1718, 137 | 2521 | 17 | 1, 2470, 2297, 38, 1965 |
| 2557 | 2 | 1, 2374, 2381, 1778, 2085 | 2593 | 7 | 1, 1160, 2165, 1264, 963 |
| 2617 | 5 | 1, 686, 1630, 1485, 749 | 2677 | 2 | 1, 515, 994, 812, 279 |
| 2689 | 19 | 1267, 322, 965, 1946, 489 | 2713 | 5 | 1, 338, 27, 569, 1678 |
| 2749 | 6 | 1, 2080, 219, 1118, 1793 | 2797 | 2 | 1, 1853, 561, 1868, 2080 |
| 2809 | * | 1, 1406, 2625, 304, 587 | 2833 | 5 | 1, 2344, 1379, 1677, 1250 |
| 2857 | 11 | 931, 1943, 308, 1989, 1348 | 2917 | 5 | 1, 2537, 208, 1827, 2108 |
| 2953 | 13 | 1, 1269, 2447, 2620, 1268 | 3001 | 14 | 145, 1954, 2895, 467, 1874 |
| 3037 | 2 | 1, 2252, 2247, 989, 2050 | 3049 | 11 | 1, 2049, 1960, 2336, 377 |
| 3061 | 6 | 1, 1815, 2936, 827, 430 | 3109 | 6 | 1, 112, 1707, 2504, 467 |
| 3121 | 7 | 1, 2951, 2912, 1102, 81 | 3169 | 7 | 433, 1892, 827, 1312, 723 |
| 3181 | 7 | 2095, 443, 2019, 1304, 1582 | 3217 | 5 | 1, 2698, 2751, 2513, 2996 |
| 3229 | 6 | 1, 44, 2691, 143, 2440 | 3253 | 2 | 865, 1814, 1955, 2206, 2535 |
| 3301 | 6 | 1, 866, 1925, 291, 862 | 3313 | 10 | 1, 2769, 2860, 527, 1802 |
| 3361 | 22 | 1, 2420, 197, 2752, 2505 | 3373 | 5 | 1, 2925, 2231, 568, 722 |
| 3433 | 5 | 1, 2795, 99, 1486, 692 | 3457 | 7 | 1, 2901, 820, 1127, 2114 |
| 3469 | 2 | 1, 1493, 1160, 165, 2278 | 3481 | * | 1, 3100, 3461, 2889, 2918 |
| 3517 | 2 | 1807, 3003, 3452, 46, 1709 | 3529 | 17 | 1909, 2948, 1270, 1377, 2411 |
| 3541 | 7 | 1, 794, 640, 1179, 3197 | 3613 | 2 | 1, 748, 1940, 1949, 1257 |
| 3637 | 2 | 1, 789, 2549, 3556, 1964 | 3673 | 5 | 2083, 3320, 808, 1893, 3377 |
| 3697 | 5 | 175, 536, 765, 448, 2933 | 3709 | 2 | 1, 1593, 1421, 2644, 2354 |
| 3721 | * | 1, 124, 2361, 3713, 2252 | 3733 | 2 | 1537, 1287, 692, 2668, 1829 |
| 3769 | 7 | 1, 878, 2099, 1624, 3 | 3793 | 5 | 3457, 2751, 352, 2597, 1670 |
| 3853 | 2 | 3085, 2163, 2549, 1726, 3080 | 3877 | 2 | 1, 1790, 3167, 1215, 1678 |
| 3889 | 11 | 1, 3692, 2823, 3479, 3214 | 4021 | 2 | 1, 2558, 3052, 3009, 1247 |
| 4057 | 5 | 1945, 1264, 387, 1127, 3686 | 4093 | 2 | 25, 2, 1792, 2801, 1047 |
| 4129 | 13 | 1, 1828, 3255, 425, 632 | 4153 | 5 | 1, 1299, 1415, 3178, 500 |
| 4177 | 5 | 2545, 2, 323, 705, 2662 | 4201 | 11 | 3643, 3962, 3809, 2830, 531 |
| 4261 | 2 | 1, 2, 1343, 2452, 2145 | 4273 | 5 | 3457, 680, 1198, 513, 2855 |
| 4297 | 5 | 1, 2800, 3201, 2219, 2270 | 4357 | 2 | 1, 4, 3557, 615, 1982 |
| 4441 | 21 | 1, 2919, 122, 4385, 2986 | 4489 | * | 1, 2584, 1377, 3443, 2552 |
| 4513 | 7 | 4141, 2325, 2102, 1811, 4372 | 4549 | 6 | 1, 4282, 2957, 1934, 3411 |
| 4561 | 11 | 2647, 608, 2697, 4121, 4336 | 4597 | 5 | 1, 3735, 3238, 2390, 3239 |
| 4621 | 2 | 1, 3664, 3215, 999, 1940 | 4657 | 15 | 1, 2140, 1107, 3899, 788 |
| 4729 | 17 | 319, 916, 3968, 3737, 1515 | 4789 | 2 | 1, 2, 3285, 2801, 832 |
| 4801 | 7 | 4225, 3142, 2061, 3377, 2732 | 4813 | 2 | 1, 2734, 2792, 4737, 971 |
| 4861 | 11 | 1, 3892, 1193, 429, 4586 | 4909 | 6 | 1, 3657, 1960, 1982, 4511 |
| 4933 | 2 | 3385, 1445, 471, 310, 500 | 4957 | 2 | 949, 278, 1828, 255, 4457 |
| 4969 | 11 | 1, 4172, 2975, 3183, 3352 | 4993 | 5 | 1, 3094, 3896, 2951, 4479 |
| 5041 | * | 139, 2680, 2025, 2876, 173 | 5077 | 2 | 4363, 2, 1715, 4828, 1323 |
| 5101 | 6 | 1, 1317, 1769, 4634, 100 | 5113 | 19 | 1, 2973, 3764, 232, 5051 |
| 5197 | 7 | 3379, 3987, 4619, 494, 1990 | 5209 | 17 | 4837, 604, 4217, 2217, 2030 |
| 5233 | 10 | 1, 832, 968, 687, 5063 | 5281 | 7 | 1, 188, 4061, 2524, 2877 |
| 5329 | * | 1, 1184, 2555, 4029, 4156 | 5413 | 5 | 1, 2631, 4232, 1883, 2632 |
| 5437 | 5 | 1, 232, 3008, 3165, 2021 | 5449 | 7 | 1, 760, 1793, 404, 2787 |
| 5521 | 11 | 3937, 3692, 1384, 2339, 489 | 5557 | 2 | 1, 880, 4976, 1097, 345 |
| 5569 | 13 | 1, 190, 2948, 2235, 1055 | 5581 | 6 | 1, 586, 3513, 1067, 1694 |
| 5641 | 14 | 5485, 3916, 2991, 2630, 2939 | 5653 | 5 | 1, 3117, 1678, 1199, 2480 |
| 5689 | 11 | 1, 136, 1697, 639, 2282 | 5701 | 2 | 1, 3557, 188, 1006, 489 |
| 5737 | 5 | 1777, 4415, 4971, 1600, 5282 | 5749 | 2 | 1, 5661, 1186, 779, 1424 |
| 5821 | 6 | 1, 2685, 4540, 4892, 4709 | 5857 | 7 | 427, 4034, 5650, 1499, 5307 |
| 5869 | 2 | 1, 2, 2313, 4384, 4415 | 5881 | 31 | 2425, 1654, 686, 4481, 4497 |
| 5953 | 7 | 1, 1743, 2650, 3956, 1511 | 6037 | 5 | 1, 1485, 3440, 1175, 868 |
| 6073 | 10 | 1, 3622, 1227, 5987, 5090 | 6121 | 7 | 1, 124, 993, 3716, 413 |
| 6133 | 5 | 1, 891, 1270, 3170, 5099 | 6217 | 5 | 1, 404, 4385, 2271, 1006 |
| 6229 | 2 | 1, 2, 4199, 1240, 3321 | 6241 | * | 1, 80, 1593, 4937, 3154 |
| 6277 | 2 | 1, 2, 5824, 3267, 4649 | 6301 | 10 | 1, 1556, 4155, 4937, 3574 |
| 6337 | 10 | 1, 836, 2067, 4079, 280 | 6361 | 19 | 4855, 4796, 5253, 2434, 1289 |
| 6373 | 2 | 1, 1258, 2907, 4874, 881 | 6397 | 2 | 1, 6074, 4727, 3867, 1150 |
| 6421 | 6 | 1, 5661, 4244, 449, 2614 | 6469 | 2 | 1, 2, 3831, 3376, 563 |
| 6481 | 7 | 1, 4096, 1629, 2036, 3305 | 6529 | 7 | 1, 5768, 6208, 5997, 6263 |
| 6553 | 10 | 1, 392, 2873, 3322, 2559 | 6577 | 5 | 1, 898, 1622, 5031, 887 |
| 6637 | 2 | 3289, 4095, 2606, 712, 2129 | 6661 | 6 | 1, 1497, 4844, 4798, 3599 |
| 6673 | 5 | 1, 4136, 3071, 3357, 586 | 6709 | 2 | 847, 2, 4665, 5237, 5728 |
| 6733 | 2 | 1, 3572, 1216, 795, 683 | 6781 | 2 | 1, 1016, 5811, 352, 4763 |
| 6793 | 10 | 1, 892, 3839, 4125, 950 | 6829 | 2 | 1, 5482, 5324, 3597, 5585 |
| 6841 | 22 | 1, 5764, 3201, 3803, 5642 | 6889 | * | 1, 3446, 5283, 4175, 4906 |
| 6949 | 2 | 1, 128, 6017, 5026, 1473 | 6961 | 13 | 1, 6320, 2717, 6129, 3862 |
| 6997 | 5 | 1, 6928, 3905, 1863, 5594 | 7057 | 5 | 1, 1942, 4613, 1040, 4353 |
| 7069 | 2 | 6493, 845, 4448, 2997, 1372 | 7129 | 7 | 1543, 4108, 2876, 473, 5421 |
| 7177 | 10 | 1, 1964, 3736, 6153, 5831 | 7213 | 5 | 1, 3861, 1618, 4598, 617 |

Table 13: Table of $\gamma_1, \ldots, \gamma_5$ for RBIBD$(11q+1, 12, 1)$ construction (cont.)

| $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ | $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ |
|---|---|---|---|---|---|
| 7237 | 2 | 1, 2, 2134, 2537, 1911 | 7297 | 5 | 1375, 3458, 1930, 6143, 5889 |
| 7309 | 6 | 1, 4907, 898, 4700, 3753 | 7321 | 7 | 1, 5764, 4197, 3548, 5825 |
| 7333 | 6 | 1, 4137, 941, 4552, 5504 | 7369 | 7 | 7051, 1263, 1526, 2692, 4943 |
| 7393 | 5 | 1, 4498, 4943, 4778, 5529 | 7417 | 5 | 5653, 1490, 987, 7223, 5278 |
| 7477 | 2 | 1, 6592, 6767, 5025, 2450 | 7489 | 7 | 1, 1520, 5038, 3365, 4587 |
| 7537 | 7 | 1, 6267, 6215, 3376, 1574 | 7549 | 2 | 1, 1786, 2703, 2840, 1721 |
| 7561 | 13 | 1, 458, 1619, 3015, 6616 | 7573 | 2 | 3421, 2, 1959, 5519, 76 |
| 7621 | 2 | 1, 3088, 254, 3323, 945 | 7669 | 2 | 7645, 2, 6417, 7000, 4619 |
| 7681 | 17 | 3517, 3934, 569, 1628, 5679 | 7717 | 2 | 1237, 2283, 941, 3946, 7352 |
| 7741 | 7 | 1, 4955, 7148, 514, 7311 | 7753 | 10 | 1921, 6206, 5854, 3555, 1421 |
| 7789 | 2 | 1, 6980, 7233, 2237, 4318 | 7873 | 5 | 1, 4400, 4924, 7773, 3863 |
| 7921 | * | 1, 4049, 5536, 3638, 6423 | 7933 | 2 | 1, 7618, 737, 2504, 135 |
| 7993 | 5 | 1, 6844, 6851, 1808, 2697 | 8017 | 5 | 1, 7496, 1961, 5866, 3873 |
| 8053 | 2 | 5449, 2771, 3478, 2727, 6224 | 8089 | 17 | 5353, 1672, 795, 6782, 7181 |
| 8101 | 6 | 1, 4131, 484, 1592, 2225 | 8161 | 7 | 1, 6454, 2735, 338, 1653 |
| 8209 | 7 | 1, 4732, 2573, 1334, 2811 | 8221 | 2 | 5995, 2, 6443, 6358, 3243 |
| 8233 | 10 | 1, 6124, 1631, 4802, 6207 | 8269 | 2 | 1, 2, 8050, 4281, 4229 |
| 8293 | 2 | 1, 3910, 2162, 7491, 2729 | 8317 | 6 | 1, 7911, 500, 3197, 4180 |
| 8329 | 7 | 1903, 4246, 6188, 5243, 7797 | 8353 | 5 | 1, 868, 5858, 8261, 141 |
| 8377 | 5 | 199, 2813, 7341, 3032, 994 | 8389 | 6 | 1, 5235, 989, 5518, 6200 |
| 8461 | 6 | 1, 1490, 40, 2451, 467 | 8521 | 13 | 1, 5282, 1853, 4060, 2283 |
| 8581 | 6 | 1, 7358, 3861, 3814, 3305 | 8629 | 6 | 1, 3483, 8468, 1967, 1372 |
| 8641 | 17 | 1, 6502, 4083, 8450, 7907 | 8677 | 2 | 6565, 2, 730, 4833, 5801 |
| 8689 | 13 | 1, 8086, 647, 2078, 3951 | 8713 | 5 | 1, 7594, 5624, 5681, 1323 |
| 8737 | 5 | 1, 7118, 6209, 8649, 5122 | 8761 | 23 | 541, 3484, 200, 5069, 5607 |
| 8821 | 2 | 1, 2, 268, 8439, 3557 | 8893 | 5 | 1, 2793, 4130, 7822, 7505 |
| 8929 | 11 | 1, 4030, 7361, 5373, 5336 | 8941 | 6 | 1, 8007, 5180, 3484, 7109 |
| 9001 | 7 | 1, 5152, 6519, 7058, 449 | 9013 | 5 | 1, 2007, 1222, 725, 8396 |
| 9049 | 7 | 1, 4762, 5943, 3653, 8624 | 9109 | 10 | 1, 6604, 8396, 9039, 1835 |
| 9133 | 6 | 1, 1821, 7961, 1340, 6688 | 9157 | 6 | 1, 3038, 7731, 5471, 8092 |
| 9181 | 2 | 2557, 452, 6201, 4996, 3341 | 9241 | 13 | 1, 1724, 9197, 5931, 6820 |
| 9277 | 5 | 1, 4041, 7637, 748, 1184 | 9337 | 5 | 811, 3788, 3556, 4641, 491 |
| 9349 | 2 | 1, 6562, 1065, 446, 7751 | 9397 | 2 | 1, 2822, 550, 7071, 4415 |
| 9409 | * | 7711, 3332, 3358, 6725, 1899 | 9421 | 2 | 7621, 2, 9081, 4132, 797 |
| 9433 | 5 | 1, 9386, 658, 5933, 1203 | 9601 | 13 | 487, 4052, 9412, 7775, 759 |
| 9613 | 2 | 1489, 2, 5271, 310, 857 | 9649 | 7 | 1, 5240, 4587, 881, 6472 |
| 9661 | 2 | 1, 5992, 5252, 6617, 6465 | 9697 | 10 | 1, 3818, 3059, 339, 208 |
| 9721 | 7 | 1, 7846, 1421, 7413, 2000 | 9733 | 2 | 1, 4322, 1641, 8459, 6400 |
| 9769 | 13 | 1, 1600, 5612, 1019, 7893 | 9781 | 6 | 1, 8475, 7508, 2350, 4859 |
| 9817 | 5 | 9091, 4443, 5830, 5894, 8681 | 9829 | 10 | 8407, 6986, 4702, 3845, 1497 |
| 9901 | 2 | 9133, 2, 7073, 3873, 8800 | 9949 | 2 | 4159, 2, 8321, 993, 448 |
| 9973 | 11 | 5611, 3093, 9080, 598, 8957 | 10009 | 11 | 1, 7316, 1401, 2722, 3941 |
| 10069 | 2 | 1, 2, 9969, 2854, 4457 | 10093 | 2 | 1, 6584, 2373, 6526, 8309 |
| 10141 | 2 | 1, 4030, 6777, 5642, 1019 | 10177 | 7 | 1, 7840, 7364, 6545, 8559 |
| 10201 | * | 1, 5201, 6704, 6357, 682 | 10273 | 10 | 1, 7220, 4689, 6922, 7889 |
| 10321 | 7 | 1, 7754, 1462, 9093, 4103 | 10333 | 5 | 1, 3069, 3184, 7643, 3662 |
| 10357 | 2 | 6805, 6860, 9521, 6298, 10335 | 10369 | 13 | 1, 1288, 3632, 3383, 7683 |
| 10429 | 7 | 8239, 7118, 3449, 586, 9381 | 10453 | 5 | 325, 9182, 7942, 4151, 6171 |
| 10477 | 2 | 2185, 2, 3875, 6699, 6436 | 10501 | 2 | 1, 7214, 10145, 3610, 5847 |
| 10513 | 7 | 8317, 10300, 5978, 5567, 8601 | 10597 | 5 | 1, 8271, 7829, 572, 2584 |
| 10609 | * | 1, 9152, 7312, 1961, 3687 | 10657 | 7 | 1, 9501, 3500, 8213, 9712 |
| 10729 | 7 | 1, 10004, 5758, 3059, 10677 | 10753 | 11 | 1, 5499, 1528, 7184, 2345 |
| 10789 | 2 | 1, 9370, 1442, 9209, 9879 | 10837 | 2 | 1, 6070, 10661, 10358, 7971 |
| 10861 | 2 | 1, 2, 8195, 562, 1167 | 10909 | 2 | 1, 6082, 8987, 1832, 3765 |
| 10957 | 5 | 1, 1755, 10048, 9701, 386 | 10993 | 7 | 10471, 8420, 10227, 5674, 5447 |
| 11113 | 13 | 1, 393, 6844, 1613, 3098 | 11149 | 10 | 1, 4029, 7682, 10691, 58 |
| 11161 | 7 | 1, 550, 9392, 791, 3735 | 11173 | 5 | 1, 9987, 2978, 10048, 9473 |
| 11197 | 2 | 3067, 6250, 2300, 7337, 4035 | 11257 | 10 | 7303, 3272, 2537, 9382, 7803 |
| 11317 | 2 | 1, 10268, 11001, 9497, 6082 | 11329 | 7 | 1, 6988, 3428, 7509, 10997 |
| 11353 | 7 | 1765, 10726, 10013, 7802, 8931 | 11437 | 2 | 1, 7472, 2321, 6172, 6525 |
| 11449 | * | 1, 6927, 6152, 8884, 6581 | 11497 | 7 | 6043, 830, 3639, 7355, 6376 |
| 11593 | 5 | 1, 9860, 9599, 9148, 5787 | 11617 | 10 | 1, 6992, 11176, 1683, 7931 |
| 11677 | 2 | 11125, 8846, 11608, 2853, 725 | 11689 | 7 | 1, 8396, 5896, 2099, 1917 |
| 11701 | 6 | 1, 2165, 7785, 6208, 11336 | 11821 | 2 | 2023, 2487, 10178, 9839, 7072 |
| 11833 | 5 | 1, 1366, 3171, 11348, 9551 | 11881 | * | 1, 8360, 1149, 11051, 11644 |
| 11941 | 10 | 5575, 11150, 11782, 1389, 8141 | 11953 | 5 | 1, 11494, 2253, 9728, 8831 |
| 12037 | 5 | 3043, 5084, 3179, 2932, 6987 | 12049 | 13 | 1, 10048, 3993, 8717, 10760 |
| 12073 | 7 | 1, 1480, 41, 7857, 4634 | 12097 | 5 | 1, 50, 10761, 94, 8597 |
| 12109 | 6 | 1, 10953, 4043, 7312, 950 | 12157 | 2 | 1, 11924, 9970, 8391, 7163 |
| 12241 | 7 | 1, 1994, 832, 1737, 8717 | 12253 | 2 | 1, 11614, 10281, 10805, 10232 |
| 12277 | 2 | 1, 7462, 2415, 5987, 4106 | 12289 | 11 | 109, 4172, 11391, 8206, 11471 |
| 12301 | 2 | 1, 2, 11734, 2607, 11957 | 12373 | 2 | 1, 962, 7427, 9939, 11662 |
| 12409 | 7 | 10861, 6376, 2885, 11090, 10275 | 12421 | 7 | 4093, 9155, 6016, 9584, 4251 |
| 12433 | 13 | 1, 6362, 5907, 5302, 5231 | 12457 | 10 | 4225, 3400, 1905, 6356, 6401 |
| 12517 | 6 | 1, 10113, 8716, 10997, 3518 | 12541 | 14 | 4735, 3332, 10397, 6256, 9819 |
| 12553 | 5 | 1, 7852, 10019, 8361, 7826 | 12577 | 10 | 1, 4880, 4864, 5361, 8297 |
| 12589 | 2 | 1, 38, 657, 10762, 6965 | 12601 | 11 | 8671, 10718, 6358, 12431, 5145 |
| 12613 | 2 | 1, 6622, 12093, 7598, 10037 | 12637 | 2 | 1, 11134, 6218, 5085, 12263 |
| 12697 | 7 | 9967, 2468, 7595, 11542, 4005 | 12721 | 13 | 1, 1570, 9651, 857, 11528 |
| 12757 | 2 | 8617, 2, 7505, 10438, 4857 | 12769 | * | 1, 6497, 8522, 9357, 6184 |
| 12781 | 2 | 1, 3578, 4745, 5044, 8361 | 12829 | 2 | 1, 6944, 4, 10145, 12789 |
| 12841 | 21 | 1, 9590, 7655, 7551, 8812 | 12853 | 5 | 1, 11871, 5057, 12442, 1922 |
| 12889 | 13 | 1, 9982, 4725, 2915, 8180 | 12973 | 14 | 1, 237, 12238, 10664, 2567 |
| 13009 | 7 | 1, 2582, 3491, 8548, 4269 | 13033 | 5 | 11863, 5006, 6892, 5189, 5697 |
| 13093 | 6 | 1, 5931, 5932, 173, 7952 | 13177 | 5 | 1, 11770, 5831, 7682, 10761 |
| 13249 | 7 | 1, 11582, 609, 11218, 2711 | 13297 | 5 | 12055, 11794, 6509, 12243, 12086 |
| 13309 | 6 | 1, 639, 6293, 7172, 9580 | 13381 | 10 | 7855, 2330, 586, 3935, 7941 |
| 13417 | 5 | 1, 10640, 681, 11686, 10811 | 13441 | 11 | 1, 4600, 8375, 1125, 4022 |
| 13477 | 2 | 9085, 2, 12820, 8343, 9995 | 13513 | 5 | 1, 7415, 10028, 10354, 9627 |
| 13537 | 7 | 10243, 9542, 5855, 10665, 7444 | 13597 | 5 | 1, 723, 4037, 5978, 6196 |
| 13633 | 5 | 1, 2702, 5999, 238, 1515 | 13669 | 6 | 1, 113, 2650, 4083, 7742 |
| 13681 | 22 | 8575, 12352, 7767, 11159, 3458 | 13693 | 6 | 1, 7041, 518, 4930, 4037 |
| 13729 | 23 | 1, 13240, 7221, 2966, 12149 | 13789 | 7 | 5845, 10700, 6262, 513, 11525 |
| 13873 | 5 | 1, 3838, 12161, 12237, 10316 | 13921 | 7 | 1, 316, 7317, 5717, 8276 |
| 13933 | 2 | 1, 2800, 1613, 9728, 8433 | 14029 | 6 | 1, 13263, 11174, 1727, 5836 |
| 14149 | 6 | 1, 2205, 6644, 5062, 8819 | 14173 | 2 | 11845, 2, 11848, 7401, 9215 |
| 14197 | 11 | 8287, 11355, 7076, 1673, 13270 | 14221 | 2 | 7951, 10622, 10594, 4449, 4427 |
| 14281 | 19 | 1, 12674, 10919, 3375, 14218 | 14293 | 6 | 11359, 12825, 2248, 6026, 11999 |
| 14341 | 2 | 9931, 2, 1203, 7469, 3478 | 14389 | 2 | 1, 3196, 9395, 13291, 986 |
| 14401 | 11 | 1, 2966, 7383, 10816, 13601 | 14437 | 5 | 1, 1689, 13379, 14314, 8726 |
| 14449 | 22 | 1, 3956, 7576, 4841, 7965 | 14461 | 2 | 1, 12488, 2788, 2613, 8105 |
| 14533 | 2 | 4123, 2, 5745, 12790, 9881 | 14557 | 2 | 2107, 2, 706, 13655, 14451 |
| 14593 | 5 | 1, 376, 11415, 7646, 689 | 14629 | 2 | 13699, 3, 13300, 10985, 2024 |
| 14641 | * | 1, 8781, 9004, 101, 5048 | 14653 | 2 | 1, 3092, 3435, 8572, 5765 |
| 14713 | 5 | 7525, 13996, 6809, 237, 3998 | 14737 | 10 | 1, 10912, 1785, 2483, 9980 |
| 14797 | 2 | 1, 11866, 11438, 1371, 14111 | 14821 | 2 | 6193, 2, 6934, 14415, 9851 |
| 14869 | 2 | 1, 6308, 5381, 6189, 13132 | 14929 | 7 | 1, 11116, 5921, 2294, 507 |
| 15013 | 2 | 1, 11104, 809, 8612, 11577 | 15061 | 2 | 1, 13306, 2066, 9677, 8763 |
| 15073 | 5 | 1, 2342, 11585, 6730, 12975 | 15121 | 11 | 1, 952, 8651, 9320, 11667 |
| 15193 | 5 | 1, 3362, 7323, 5320, 5933 | 15217 | 10 | 8599, 12508, 11201, 171, 5960 |
| 15241 | 11 | 4711, 10768, 4910, 12345, 5711 | 15277 | 6 | 13699, 1173, 9416, 11015, 7966 |
| 15289 | 11 | 1, 2422, 10628, 7671, 15257 | 15313 | 5 | 1, 13526, 2855, 2313, 4306 |
| 15349 | 2 | 1, 2, 2357, 12945, 7654 | 15361 | 7 | 1, 9872, 9783, 1775, 3286 |

Table 13: Table of $\gamma_1, \ldots, \gamma_5$ for RBIBD$(11q+1, 12, 1)$ construction (cont.)

| $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ | $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ |
|---|---|---|---|---|---|
| 15373 | 2 | 1, 3592, 572, 11411, 9159 | 15493 | 5 | 1, 3033, 3436, 161, 10268 |
| 15541 | 6 | 1, 11810, 9189, 1666, 7157 | 15601 | 23 | 1, 5374, 12056, 15243, 1991 |
| 15625 | * | 1, 11714, 2919, 5783, 4030 | 15649 | 11 | 1, 2720, 13510, 12225, 13433 |
| 15661 | 2 | 1, 6602, 8909, 9430, 669 | 15733 | 6 | 1, 2955, 962, 3988, 7997 |
| 15817 | 5 | 1, 1052, 7240, 5391, 581 | 15877 | 5 | 1, 10094, 5013, 10931, 12538 |
| 15889 | 21 | 1735, 2925, 7442, 11459, 8176 | 15901 | 10 | 1, 9111, 2296, 6194, 8987 |
| 15913 | 5 | 10399, 10305, 4072, 2429, 2510 | 15937 | 7 | 1, 10641, 575, 11120, 2446 |
| 15973 | 7 | 5845, 5979, 2168, 5693, 3718 | 16033 | 5 | 4669, 11842, 6854, 13407, 10481 |
| 16057 | 7 | 1, 663, 15773, 12448, 8450 | 16069 | 2 | 1, 12688, 7739, 6704, 3597 |
| 16129 | * | 1, 9344, 7385, 598, 6441 | 16141 | 6 | 1, 8495, 315, 9118, 902 |
| 16189 | 2 | 1, 6140, 12076, 4419, 6041 | 16249 | 17 | 1, 11372, 14477, 448, 5493 |
| 16273 | 7 | 1, 4628, 2001, 184, 7301 | 16333 | 2 | 1, 14950, 15, 7325, 8666 |
| 16369 | 7 | 1, 2384, 15393, 5303, 9448 | 16381 | 2 | 1, 14, 6731, 4545, 6412 |
| 16417 | 10 | 1, 686, 10197, 10000, 2669 | 16453 | 2 | 1, 4082, 8164, 6879, 4229 |
| 16477 | 2 | 10039, 2, 13481, 2194, 10497 | 16561 | 7 | 1, 14656, 4364, 5571, 12119 |
| 16573 | 2 | 5293, 10796, 7295, 5428, 15999 | 16633 | 15 | 1, 1640, 14866, 8649, 281 |
| 16657 | 5 | 9163, 3088, 6429, 9098, 16067 | 16693 | 2 | 7279, 2, 885, 7145, 8518 |
| 16729 | 13 | 1, 4898, 2003, 13245, 6634 | 16741 | 6 | 1, 8661, 3158, 9869, 4018 |
| 16921 | 17 | 13813, 12920, 15892, 10071, 14831 | 16981 | 2 | 1, 8512, 13427, 6158, 8139 |
| 16993 | 10 | 1, 6796, 7508, 10817, 15981 | 17029 | 10 | 1, 8782, 3746, 1115, 3219 |
| 17041 | 7 | 1, 11060, 12281, 13300, 381 | 17053 | 2 | 3253, 2, 15754, 9861, 3029 |
| 17077 | 2 | 1, 6038, 10071, 5512, 7079 | 17137 | 5 | 1, 16760, 15760, 11249, 16635 |
| 17161 | * | 1, 14552, 3100, 743, 8763 | 17209 | 14 | 1, 12448, 10886, 6197, 2853 |
| 17257 | 5 | 1, 6658, 13539, 4748, 5783 | 17293 | 7 | 5863, 9489, 7738, 14282, 6011 |
| 17317 | 2 | 2353, 2, 322, 9443, 12657 | 17341 | 6 | 1, 5409, 14774, 208, 4139 |
| 17377 | 7 | 1, 8428, 3407, 1004, 15147 | 17389 | 2 | 1, 2, 2854, 16577, 3381 |
| 17401 | 11 | 1, 2228, 17015, 1138, 13593 | 17449 | 14 | 1, 15778, 3191, 13250, 63 |
| 17497 | 5 | 1, 7516, 3665, 7892, 9843 | 17509 | 2 | 1, 10294, 12896, 1955, 16605 |
| 17569 | 11 | 1, 15652, 4331, 9722, 549 | 17581 | 10 | 3955, 7910, 2661, 731, 4936 |
| 17713 | 7 | 1, 15436, 2990, 2715, 6371 | 17737 | 7 | 1, 4792, 11607, 7442, 2417 |
| 17749 | 2 | 10633, 2, 2669, 184, 5637 | 17761 | 19 | 1, 17426, 8999, 2062, 1533 |
| 17881 | 7 | 7687, 2056, 15212, 10773, 3515 | 17929 | 11 | 1, 16508, 9166, 3911, 8901 |
| 17977 | 5 | 1, 5576, 16018, 4529, 1977 | 17989 | 2 | 1, 2120, 2092, 14771, 12927 |
| 18013 | 2 | 1, 7430, 7739, 8205, 1990 | 18049 | 13 | 1, 10544, 8709, 8590, 16757 |
| 18061 | 6 | 1, 15915, 8926, 12020, 5873 | 18097 | 5 | 12709, 14114, 16283, 5379, 1930 |
| 18121 | 23 | 11089, 2092, 12461, 12758, 16659 | 18133 | 5 | 1, 10671, 6472, 11612, 9713 |
| 18169 | 11 | 1, 3140, 2303, 11002, 3147 | 18181 | 2 | 1, 14168, 7018, 15087, 12995 |
| 18217 | 7 | 1, 14410, 17294, 2303, 13581 | 18229 | 2 | 8533, 2, 11848, 15671, 4065 |
| 18253 | 5 | 1, 10185, 12074, 13384, 12155 | 18289 | 13 | 1, 14810, 10323, 13906, 17921 |
| 18301 | 6 | 1, 16839, 11828, 13163, 15250 | 18313 | 10 | 9547, 15884, 8722, 3425, 16113 |
| 18397 | 6 | 6601, 16486, 2271, 17315, 12674 | 18433 | 5 | 1, 6296, 14632, 4815, 5615 |
| 18457 | 5 | 1, 15548, 15118, 14975, 459 | 18481 | 13 | 6403, 3824, 1288, 15867, 5771 |
| 18493 | 2 | 10771, 3, 5978, 3592, 15569 | 18517 | 6 | 1, 1599, 18226, 4508, 6773 |
| 18541 | 6 | 1, 7767, 14762, 10523, 190 | 18553 | 5 | 1, 15734, 3514, 3593, 9369 |
| 18637 | 2 | 3661, 221, 7544, 14722, 3453 | 18661 | 10 | 1, 12338, 10119, 14843, 8206 |
| 18757 | 2 | 1, 1792, 7325, 18272, 3021 | 18769 | * | 1, 9521, 3028, 18177, 506 |
| 18793 | 5 | 1, 358, 3669, 18707, 18140 | 18913 | 7 | 511, 9290, 3166, 8963, 17445 |
| 18973 | 2 | 1, 380, 16336, 17909, 10023 | 19009 | 23 | 1, 15620, 5146, 14771, 14523 |
| 19069 | 2 | 1, 13610, 16895, 11932, 16029 | 19081 | 17 | 1, 10737, 1330, 5606, 14585 |
| 19141 | 2 | 1, 1912, 11381, 16544, 4419 | 19213 | 5 | 1, 10821, 12340, 13199, 8636 |
| 19237 | 2 | 1, 17590, 5793, 6881, 1502 | 19249 | 7 | 17905, 5222, 3, 7727, 3316 |
| 19273 | 5 | 18127, 2404, 6563, 285, 12404 | 19309 | 6 | 1, 15423, 4886, 9244, 15077 |
| 19321 | * | 1, 140, 13847, 14272, 14535 | 19333 | 2 | 1, 13490, 1024, 10773, 4211 |
| 19381 | 7 | 11677, 5315, 17973, 7216, 18158 | 19417 | 5 | 1, 18424, 2012, 4481, 18915 |
| 19429 | 6 | 5743, 8079, 3592, 12839, 9158 | 19441 | 13 | 1, 4264, 5153, 356, 14187 |
| 19477 | 6 | 5443, 6861, 8717, 14786, 6778 | 19489 | 19 | 5437, 15676, 3167, 19124, 8601 |
| 19501 | 2 | 1, 10852, 10424, 4043, 17937 | 19597 | 2 | 1, 3586, 17234, 15971, 1689 |
| 19609 | 13 | 9019, 3680, 13583, 6345, 11188 | 19681 | 11 | 6391, 7420, 8084, 4211, 5667 |
| 19717 | 2 | 1, 6140, 3171, 6964, 15107 | 19753 | 5 | 1, 11984, 18891, 178, 14657 |
| 19777 | 11 | 1, 8325, 2734, 14660, 17975 | 19801 | 13 | 1, 15560, 10457, 14836, 14133 |
| 19813 | 2 | 67, 2, 14626, 18389, 8979 | 19861 | 11 | 1, 5073, 11888, 12413, 3136 |
| 19993 | 10 | 1, 18056, 1965, 10355, 7594 | 20029 | 2 | 1, 18508, 18369, 18176, 15869 |
| 20089 | 7 | 6487, 9188, 11043, 10511, 13660 | 20101 | 6 | 1, 16719, 13568, 7127, 808 |
| 20113 | 10 | 3013, 9706, 6731, 16826, 15807 | 20149 | 2 | 2209, 17043, 2932, 14435, 14480 |
| 20161 | 13 | 1225, 19286, 7539, 1174, 15041 | 20173 | 2 | 16471, 5213, 12994, 3842, 3267 |
| 20233 | 5 | 1, 13595, 957, 19078, 12386 | 20269 | 2 | 6487, 2, 11812, 2315, 18153 |
| 20341 | 2 | 1, 11236, 893, 854, 10269 | 20353 | 5 | 1, 15032, 19793, 8007, 17332 |
| 20389 | 6 | 1, 14961, 18586, 14891, 17522 | 20509 | 2 | 1, 2, 15615, 15370, 95 |
| 20521 | 11 | 1, 9176, 17734, 12089, 12063 | 20533 | 2 | 835, 2, 837, 6701, 19036 |
| 20593 | 5 | 13255, 8690, 6741, 16883, 3874 | 20641 | 7 | 1, 11768, 6154, 725, 12561 |
| 20749 | 2 | 17659, 5456, 7127, 13558, 10533 | 20773 | 2 | 17143, 2, 20445, 10613, 12586 |
| 20809 | 7 | 1, 8751, 5450, 4061, 11416 | 20857 | 10 | 1, 3880, 3020, 6755, 9153 |
| 20929 | 7 | 8965, 12710, 14200, 4719, 5543 | 21001 | 11 | 1, 8192, 19427, 3243, 16906 |
| 21013 | 2 | 1, 19526, 13433, 6460, 13209 | 21061 | 7 | 12295, 18045, 7541, 20530, 6824 |
| 21121 | 19 | 17593, 1706, 15743, 1017, 14962 | 21157 | 2 | 17605, 2, 8231, 2164, 15663 |
| 21169 | 13 | 1, 18290, 2224, 3615, 17069 | 21193 | 11 | 1, 15081, 19984, 14969, 9062 |
| 21277 | 6 | 1, 14103, 3077, 20444, 21214 | 21313 | 5 | 11299, 10700, 5242, 18711, 10283 |
| 21397 | 2 | 985, 2, 18873, 8566, 15695 | 21433 | 5 | 1, 16850, 9263, 4066, 19983 |
| 21481 | 13 | 1, 9742, 212, 5277, 18437 | 21493 | 2 | 1, 5720, 6231, 4426, 2279 |
| 21517 | 5 | 1, 5829, 12400, 15401, 7460 | 21529 | 11 | 1, 10156, 1433, 16112, 9195 |
| 21577 | 5 | 18757, 12460, 4467, 9101, 12650 | 21589 | 2 | 4573, 2, 3029, 2158, 20817 |
| 21601 | 7 | 1, 16888, 6617, 16544, 21339 | 21613 | 2 | 1, 15104, 15370, 17169, 287 |
| 21649 | 14 | 1, 19786, 3135, 17198, 9737 | 21661 | 2 | 1, 14740, 15200, 12935, 6345 |
| 21673 | 10 | 11875, 14456, 3910, 21027, 18521 | 21757 | 5 | 14581, 5368, 16677, 21734, 13385 |
| 21817 | 7 | 1, 12268, 20645, 16611, 2696 | 21841 | 11 | 1, 20860, 19749, 17573, 3740 |
| 21937 | 7 | 1, 18436, 104, 5261, 5673 | 21961 | 17 | 1795, 2116, 14912, 2199, 20159 |
| 21997 | 7 | 14041, 11559, 19906, 12824, 15419 | 22093 | 6 | 1, 8541, 17957, 17282, 18574 |
| 22129 | 19 | 1, 4874, 9029, 5026, 18837 | 22153 | 5 | 1, 3850, 4011, 20510, 7019 |
| 22189 | 2 | 1, 7724, 16882, 17591, 11073 | 22201 | * | 1, 11249, 1624, 20144, 11739 |
| 22273 | 5 | 1, 19544, 13019, 1132, 6969 | 22369 | 11 | 1, 17672, 13006, 13037, 18555 |
| 22381 | 10 | 1, 16025, 8384, 9027, 17578 | 22441 | 14 | 1, 10280, 8356, 10463, 135 |
| 22453 | 5 | 1, 4383, 2236, 13640, 18659 | 22501 | 2 | 1, 13462, 12956, 11699, 17061 |
| 22549 | 2 | 1, 2, 10810, 5, 12015 | 22573 | 6 | 1, 11153, 20848, 20187, 16532 |
| 22621 | 2 | 1, 4522, 13749, 21800, 5771 | 22669 | 2 | 9931, 2, 9933, 19156, 3371 |
| 22717 | 2 | 2947, 1622, 11297, 20823, 16078 | 22741 | 7 | 1, 16178, 12965, 2451, 7354 |
| 22777 | 7 | 20209, 17721, 4240, 21428, 10949 | 22801 | * | 19897, 6080, 1936, 7493, 10293 |
| 22861 | 2 | 17731, 2, 4, 6929, 8571 | 22921 | 7 | 12757, 17522, 10425, 2110, 17369 |
| 22993 | 5 | 1, 18682, 20870, 17735, 15417 | 23017 | 5 | 1, 21578, 9340, 14171, 20913 |
| 23029 | 2 | 1, 8038, 18173, 4838, 5859 | 23041 | 11 | 12643, 19196, 13581, 1631, 9886 |
| 23053 | 2 | 15769, 2, 13373, 15964, 19737 | 23173 | 5 | 1, 14871, 3874, 5249, 19454 |
| 23197 | 2 | 17299, 2, 6154, 593, 4785 | 23209 | 31 | 1, 21868, 22772, 16211, 18885 |
| 23269 | 6 | 1, 22251, 9134, 21767, 10366 | 23293 | 5 | 1, 4199, 3183, 18302, 8554 |
| 23473 | 5 | 1069, 8873, 3759, 8386, 19310 | 23497 | 5 | 1, 1162, 18747, 13277, 2012 |
| 23509 | 2 | 1, 3712, 6069, 7544, 7571 | 23557 | 5 | 7423, 1220, 4049, 2043, 46 |
| 23581 | 6 | 1, 741, 9407, 17342, 17794 | 23593 | 5 | 14611, 3658, 7808, 2919, 15047 |
| 23629 | 2 | 1, 19670, 11559, 6641, 7978 | 23677 | 5 | 1, 4185, 1253, 23540, 6112 |
| 23689 | 11 | 4675, 14126, 16137, 8573, 1138 | 23761 | 7 | 7303, 10520, 21059, 8662, 23403 |
| 23773 | 5 | 1, 21261, 2458, 21065, 5036 | 23833 | 5 | 10393, 2325, 23708, 575, 22900 |
| 23857 | 5 | 1, 18464, 16655, 2025, 15886 | 23869 | 2 | 2707, 3, 20714, 7547, 14884 |
| 23893 | 5 | 6061, 19268, 22786, 17729, 5307 | 23917 | 2 | 1, 12592, 32, 11897, 9783 |
| 23929 | 7 | 1, 12862, 12245, 21656, 5739 | 23977 | 5 | 1, 4822, 15111, 19805, 21056 |
| 24001 | 14 | 1, 7892, 10533, 18928, 7883 | 24049 | 19 | 1, 5878, 8816, 19305, 12161 |
| 24061 | 10 | 3241, 2283, 9410, 568, 5447 | 24097 | 5 | 1, 3206, 21333, 22355, 4780 |

Table 13: Table of $\gamma_1, \ldots, \gamma_5$ for RBIBD$(11q+1, 12, 1)$ construction (cont.)

| $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ | $q$ | $\omega$ | $\gamma_1, \ldots, \gamma_5$ |
|---|---|---|---|---|---|
| 24109 | 2 | 13309, 2, 13312, 9659, 4323 | 24121 | 13 | 1, 9304, 9305, 6932, 18279 |
| 24133 | 6 | 1, 21789, 10330, 14027, 16652 | 24169 | 11 | 1, 2710, 4322, 18107, 11475 |
| 24181 | 17 | 22345, 5319, 3062, 21448, 5273 | 24229 | 2 | 1, 13942, 22437, 2726, 8567 |
| 24337 | 5 | 12877, 21209, 20643, 19924, 11810 | 24373 | 7 | 17359, 21441, 16814, 7222, 9125 |
| 24421 | 7 | 14677, 22095, 20212, 11639, 13538 | 24469 | 14 | 11395, 4557, 7960, 21890, 12587 |
| 24481 | 11 | 7729, 15842, 15413, 4138, 6429 | 24517 | 5 | 1, 22425, 12299, 19354, 1868 |
| 24649 | * | 1, 1106, 12166, 3785, 18213 | 24697 | 5 | 1, 15586, 23024, 6905, 18753 |
| 24709 | 2 | 1, 2, 21058, 227, 20259 | 24733 | 2 | 12193, 2, 11572, 23499, 22133 |
| 24781 | 2 | 1, 2, 1589, 7959, 15688 | 24793 | 5 | 1, 15028, 20913, 458, 12875 |
| 24841 | 14 | 1, 20692, 4937, 19959, 5678 | 24877 | 5 | 1, 10347, 23686, 10361, 12314 |
| 24889 | 11 | 5047, 21154, 1779, 3842, 3221 | 25033 | 5 | 14059, 12734, 16510, 5999, 18477 |
| 25057 | 5 | 9031, 18556, 2753, 1971, 15992 | 25117 | 5 | 2899, 21746, 11596, 23153, 10851 |
| 25153 | 10 | 1, 24922, 3375, 21299, 19274 | 25189 | 2 | 1, 23740, 14607, 16415, 4688 |
| 25237 | 2 | 1801, 2, 20931, 8464, 18839 | 25261 | 7 | 6979, 16671, 23614, 10757, 1676 |
| 25309 | 13 | 1369, 20054, 15556, 11981, 14307 | 25321 | 19 | 1, 19334, 11679, 15934, 17501 |
| 25357 | 2 | 1, 18664, 24315, 23834, 227 | 25453 | 2 | 1, 24296, 9753, 12940, 18605 |
| 25537 | 10 | 1, 24914, 24802, 22289, 17685 | 25561 | 11 | 12409, 9550, 8277, 13400, 1439 |
| 25609 | 7 | 11539, 6556, 19793, 21207, 4214 | 25621 | 10 | 1, 20891, 7552, 25305, 19868 |
| 25633 | 5 | 1, 15532, 21185, 11889, 9512 | 25657 | 5 | 1, 21400, 25167, 22160, 269 |
| 25693 | 2 | 1, 23828, 5614, 23361, 18143 | 25717 | 2 | 1, 4154, 3, 8602, 7499 |
| 25741 | 6 | 1, 8145, 12700, 18326, 25337 | 25801 | 7 | 1, 16388, 22000, 4937, 8853 |
| 25849 | 7 | 15697, 19568, 24689, 9052, 15837 | 25873 | 10 | 1, 2026, 14492, 23297, 24603 |
| 25933 | 2 | 1, 9742, 25316, 6887, 20073 | 25969 | 7 | 8401, 18556, 1473, 107, 8708 |
| 25981 | 11 | 1, 1203, 23690, 17764, 12377 | 26017 | 5 | 1, 24100, 21794, 2673, 6173 |
| 26029 | 6 | 1, 10095, 16577, 194, 14338 | 26041 | 13 | 7351, 6356, 23291, 195, 21544 |
| 26053 | 2 | 1, 4256, 18359, 23626, 19989 | 26113 | 7 | 1, 404, 20267, 18981, 6838 |
| 26161 | 13 | 1, 20210, 14039, 5128, 615 | 26209 | 11 | 1, 23402, 15921, 24382, 8369 |
| 26293 | 6 | 1, 24747, 2939, 17884, 7892 | 26317 | 6 | 1, 4257, 25868, 2219, 15304 |
| 26437 | 5 | 1, 13923, 5968, 8168, 10313 | 26449 | 7 | 10531, 11926, 10301, 21225, 13844 |
| 26497 | 5 | 1, 16004, 2128, 17751, 19769 | 26557 | 2 | 1, 7102, 6321, 5702, 17387 |
| 26569 | * | 1, 9184, 17289, 21242, 8309 | 26641 | 7 | 1, 8024, 958, 1053, 13607 |
| 26701 | 22 | 1, 14385, 14315, 11668, 19898 | 26713 | 10 | 6079, 18358, 4016, 2579, 16767 |
| 26737 | 10 | 1, 19196, 18586, 1823, 9927 | 26821 | 2 | 25639, 2, 6340, 12519, 5225 |
| 26833 | 5 | 1, 5486, 9711, 6077, 9610 | 26881 | 11 | 7219, 26152, 21080, 3201, 14627 |
| 26893 | 5 | 1, 1881, 17728, 5984, 10295 | 26953 | 7 | 1, 21808, 5421, 11930, 6965 |
| 27061 | 2 | 1, 2, 20200, 26255, 18741 | 27073 | 5 | 1, 11402, 27033, 1709, 10108 |
| 27109 | 7 | 23731, 5925, 5560, 18020, 15131 | 27241 | 17 | 9385, 16580, 25792, 19647, 22331 |
| 27253 | 2 | 5665, 22814, 2170, 3987, 9089 | 27277 | 6 | 1, 24993, 25462, 25349, 24878 |
| 27337 | 5 | 17539, 12754, 11330, 10427, 3483 | 27361 | 7 | 1, 21146, 23321, 2410, 23931 |
| 27397 | 2 | 1, 7960, 17090, 1851, 23765 | 27409 | 13 | 1, 22234, 3147, 11534, 10127 |
| 27457 | 7 | 8671, 12082, 2384, 20241, 14627 | 27481 | 7 | 1, 17672, 5032, 26709, 11057 |
| 27529 | 7 | 1, 8462, 18671, 226, 16473 | 27541 | 19 | 1, 7064, 16503, 4121, 3898 |
| 27673 | 11 | 1, 9219, 27218, 22577, 5944 | 27697 | 5 | 1, 9208, 6747, 3728, 25949 |
| 27733 | 2 | 1, 18860, 4637, 2458, 795 | 27793 | 5 | 1, 2662, 14237, 14540, 4293 |
| 27817 | 5 | 1, 9784, 9195, 16067, 20396 | 27889 | * | 1, 27508, 21287, 6669, 24338 |
| 27901 | 2 | 1, 2, 22265, 21927, 5626 | 27961 | 13 | 21625, 25168, 25190, 10701, 27629 |
| 27997 | 5 | 1, 21287, 14716, 24776, 9003 | 28057 | 5 | 1, 16246, 6401, 2619, 23432 |
| 28069 | 7 | 18055, 7490, 16732, 21777, 245 | 28081 | 19 | 18115, 2258, 19444, 18771, 12197 |
| 28201 | 11 | 1, 11596, 21947, 10443, 6530 | 28297 | 5 | 1, 140, 8865, 22889, 24238 |
| 28309 | 2 | 1, 20812, 11993, 13905, 10034 | 28393 | 15 | 21415, 1509, 11830, 13439, 21452 |
| 28429 | 2 | 1, 2, 5307, 28360, 6011 | 28477 | 2 | 22141, 14458, 3651, 2723, 3554 |
| 28513 | 5 | 1, 21700, 12911, 10676, 26637 | 28537 | 5 | 27301, 27338, 6747, 12077, 22198 |
| 28549 | 2 | 1, 2, 22275, 20693, 27376 | 28561 | * | 1, 19040, 3376, 3545, 4653 |
| 28573 | 2 | 20809, 3485, 28118, 18711, 23020 | 28597 | 2 | 14131, 2, 16809, 24287, 27592 |
| 28621 | 13 | 17455, 4065, 22474, 5639, 3656 | 28657 | 5 | 1, 5584, 6933, 26156, 6371 |
| 28669 | 6 | 1, 6525, 1655, 1708, 7724 | 28729 | 22 | 1, 11522, 40, 25769, 18447 |
| 28753 | 10 | 1, 21868, 3621, 19109, 18422 | 28789 | 7 | 23095, 16149, 4730, 18340, 21605 |
| 28813 | 2 | 13957, 7348, 20667, 13148, 22241 | 28837 | 2 | 1, 8854, 8145, 12605, 10928 |
| 28909 | 2 | 26773, 3, 20152, 1832, 22025 | 28921 | 11 | 1, 4856, 13097, 16474, 1137 |
| 28933 | 2 | 1, 26564, 23103, 12556, 21173 | 29017 | 5 | 1, 6880, 4310, 18863, 17817 |
| 29077 | 2 | 1, 11452, 2727, 6890, 4745 | 29101 | 2 | 7375, 9670, 5180, 16611, 4937 |
| 29137 | 5 | 5293, 2180, 14559, 19606, 17627 | 29173 | 2 | 20887, 2, 4, 24461, 5061 |
| 29209 | 7 | 403, 21484, 2462, 18407, 18285 | 29221 | 2 | 1, 2, 4534, 1047, 20087 |
| 29269 | 6 | 1, 18399, 26579, 25810, 2 | 29389 | 2 | 1, 2, 20200, 23483, 25821 |
| 29401 | 13 | 14299, 25006, 2, 20847, 19511 | 29437 | 2 | 1, 9188, 24633, 22078, 8345 |
| 29473 | 5 | 1, 3988, 16383, 11675, 15206 | 29569 | 17 | 1, 27170, 10473, 19139, 12706 |
| 29581 | 10 | 18955, 8330, 6263, 20703, 5272 | 29629 | 7 | 23413, 603, 25804, 27842, 9329 |
| 29641 | 7 | 29125, 1718, 13785, 28043, 26842 | 29761 | 17 | 17371, 12880, 27987, 17924, 22451 |
| 29833 | 5 | 1, 14360, 18965, 25312, 8691 | 29881 | 7 | 3409, 4586, 27646, 16509, 23471 |
| 29917 | 2 | 1, 25840, 2355, 20915, 16970 | 29929 | * | 1, 15137, 13312, 22868, 13305 |
| 29989 | 2 | 6793, 2, 26374, 18707, 7575 | 30013 | 2 | 1, 13552, 8735, 746, 23079 |
| 30097 | 10 | 1, 10612, 5120, 24813, 2081 | 30109 | 2 | 1, 7946, 11441, 5241, 21178 |
| 30133 | 5 | 1, 15627, 134, 8446, 28847 | 30169 | 7 | 1, 10952, 1109, 11326, 11325 |
| 30181 | 2 | 5329, 3646, 7493, 28820, 15567 | 30241 | 11 | 1, 10472, 22121, 6118, 5163 |
| 30253 | 2 | 1, 5410, 5631, 19838, 28133 | 30313 | 5 | 1, 21214, 24326, 28929, 20459 |
| 30469 | 2 | 1, 7562, 3, 317, 15508 | 30493 | 6 | 1, 27863, 22742, 22708, 4869 |
| 30517 | 2 | 1, 24430, 911, 11415, 4184 | 30529 | 13 | 11581, 7400, 30034, 6303, 18545 |
| 30553 | 5 | 27091, 13934, 26147, 16126, 28827 | 30577 | 5 | 1, 4558, 19139, 27513, 5126 |
| 30637 | 2 | 1, 11954, 26878, 6953, 20757 | 30649 | 7 | 1, 11572, 7844, 14697, 17549 |
| 30661 | 2 | 1, 17066, 22055, 9538, 18693 | 30697 | 10 | 1, 19762, 15584, 26675, 9249 |
| 30757 | 5 | 1, 24123, 24124, 21125, 16208 | 30781 | 2 | 1, 15046, 13817, 27897, 13208 |
| 30817 | 5 | 469, 14228, 16143, 18785, 24712 | 30829 | 2 | 1, 4270, 3, 14564, 28931 |
| 30841 | 7 | 9187, 3944, 18725, 16731, 15922 | 30853 | 2 | 1, 3490, 13961, 24177, 7964 |
| 30937 | 15 | 1, 7018, 29369, 23409, 21692 | 30949 | 10 | 1, 22959, 25781, 17104, 24860 |
| 31033 | 10 | 1, 20944, 20945, 20439, 7022 | 31069 | 2 | 1, 2, 10031, 11650, 2331 |
| 31081 | 13 | 1, 22708, 2792, 13877, 5043 | 31153 | 10 | 1, 3080, 14608, 18689, 15159 |
| 31177 | 7 | 14275, 1383, 20852, 11170, 16013 | 31189 | 13 | 1, 20583, 15280, 15434, 12203 |
| 31237 | 6 | 1, 15183, 15184, 17729, 266 | 31249 | 23 | 1, 20, 17470, 12605, 9525 |
| 31321 | 7 | 5869, 12314, 29015, 9321, 6490 | 31333 | 5 | 1, 11637, 14282, 30040, 24449 |
| 31357 | 2 | 1, 6032, 6431, 8986, 20265 | 31393 | 5 | 1, 15572, 12112, 30465, 29177 |
| 31477 | 6 | 1, 25191, 4562, 28817, 31414 | 31489 | 7 | 1, 8638, 16187, 3416, 30051 |
| 31513 | 7 | 1, 31180, 16532, 21263, 14661 | 31573 | 5 | 1, 22119, 27880, 9221, 23102 |
| 31657 | 5 | 1, 8804, 6497, 14445, 31498 | 31729 | 7 | 1, 21548, 20691, 15862, 5345 |
| 31741 | 6 | 1, 1473, 2948, 2609, 3358 | 31849 | 14 | 1135, 30238, 30255, 4484, 8837 |
| 31873 | 11 | 1, 18861, 17746, 6104, 6275 | 31957 | 2 | 1, 30662, 18111, 2092, 671 |
| 31981 | 6 | 1, 8270, 29181, 14213, 19636 | 32029 | 2 | 13717, 20672, 25623, 13918, 11081 |
| 32041 | * | 1, 23373, 15395, 1816, 18506 | 32077 | 2 | 31963, 2, 5, 15670, 4245 |
| 32089 | 13 | 1, 344, 20849, 27358, 3135 | 32173 | 5 | 1, 28653, 12083, 4312, 19106 |
| 32233 | 5 | 1, 5920, 4317, 17366, 11873 | 32257 | 15 | 1, 15754, 20342, 15825, 16475 |
| 32341 | 2 | 21517, 2, 26159, 12628, 20133 | 32353 | 15 | 1, 24669, 8318, 5614, 1955 |
| 32377 | 5 | 1, 23134, 29834, 25347, 15887 | 32401 | 7 | 1, 28948, 18788, 25473, 27107 |
| 32413 | 5 | 1, 21225, 10403, 12082, 19940 | 32497 | 7 | 1, 17752, 6429, 13907, 7016 |
| 32533 | 2 | 1, 25016, 16137, 598, 7679 | 32569 | 7 | 1, 19490, 28439, 244, 30399 |
| 32653 | 2 | 1, 6118, 8408, 3125, 13449 | 32713 | 5 | 18577, 16943, 21927, 31268, 32482 |
| 32749 | 2 | 1, 2, 22456, 11855, 14619 | 32761 | * | 1, 24934, 5475, 19655, 25154 |

Table 14: Table of 3960 possible exceptions in the PBD-closure of $\{7, 13\}$

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 19 | 25 | 31 | 37 | 43 | 55 | 61 | 67 | 73 | 79 | 97 | 103 | 109 | 115 | 121 | 127 | 133 | 139 | 145 | 151 | 157 | 163 | 181 | 187 | 193 | 199 | 205 | 211 | 223 | 229 | 235 | 241 |
| 247 | 253 | 265 | 271 | 277 | 283 | 289 | 313 | 319 | 325 | 331 | 349 | 355 | 361 | 367 | 373 | 391 | 397 | 403 | 409 | 415 | 433 | 439 | 445 | 451 | 457 | 475 | 481 | 487 | | | |
| 493 | 499 | 505 | 523 | 529 | 535 | 541 | 565 | 571 | 577 | 583 | 607 | 613 | 619 | 625 | 643 | 649 | 655 | 661 | 667 | 685 | 691 | 697 | 703 | 709 | 715 | 727 | 733 | 739 | | | |
| 745 | 751 | 769 | 775 | 781 | 787 | 793 | 799 | 805 | 811 | 817 | 823 | 829 | 835 | 853 | 859 | 865 | 871 | 877 | 895 | 901 | 907 | 913 | 919 | 925 | 937 | 943 | 949 | 955 | | | |
| 961 | 979 | 985 | 991 | 997 | 1003 | 1021 | 1027 | 1033 | 1039 | 1045 | 1063 | 1069 | 1075 | 1081 | 1087 | 1111 | 1117 | 1123 | 1129 | 1135 | 1153 | 1159 | 1165 | | | | | | | | |
| 1171 | 1195 | 1201 | 1207 | 1213 | 1231 | 1237 | 1243 | 1249 | 1255 | 1273 | 1279 | 1285 | 1291 | 1297 | 1315 | 1321 | 1327 | 1333 | 1339 | 1357 | 1363 | 1369 | | | | | | | | |
| 1375 | 1381 | 1399 | 1405 | 1411 | 1417 | 1423 | 1435 | 1441 | 1447 | 1453 | 1459 | 1465 | 1489 | 1495 | 1501 | 1507 | 1525 | 1531 | 1537 | 1543 | 1549 | 1555 | | | | | | | | |
| 1567 | 1579 | 1585 | 1591 | 1609 | 1615 | 1621 | 1627 | 1633 | 1639 | 1645 | 1651 | 1657 | 1663 | 1669 | 1675 | 1693 | 1699 | 1705 | 1711 | 1717 | 1735 | 1741 | | | | | | | | |
| 1747 | 1753 | 1759 | 1765 | 1783 | 1789 | 1795 | 1801 | 1819 | 1825 | 1831 | 1837 | 1843 | 1861 | 1867 | 1873 | 1879 | 1885 | 1903 | 1909 | 1915 | 1921 | 1927 | | | | | | | | |
| 1945 | 1951 | 1957 | 1963 | 1969 | 1975 | 1987 | 1993 | 1999 | 2005 | 2011 | 2035 | 2047 | 2053 | 2077 | 2083 | 2089 | 2095 | 2119 | 2125 | 2131 | 2137 | 2155 | | | | | | | | |
| 2161 | 2167 | 2173 | 2179 | 2203 | 2209 | 2215 | 2221 | 2239 | 2245 | 2251 | 2257 | 2263 | 2281 | 2287 | 2293 | 2299 | 2305 | 2329 | 2335 | 2341 | 2347 | 2371 | | | | | | | | |
| 2377 | 2383 | 2389 | 2407 | 2413 | 2419 | 2425 | 2431 | 2455 | 2461 | 2467 | 2473 | 2491 | 2497 | 2503 | 2509 | 2515 | 2533 | 2539 | 2545 | 2551 | 2557 | 2575 | | | | | | | | |
| 2581 | 2587 | 2593 | 2599 | 2605 | 2623 | 2629 | 2635 | 2641 | 2665 | 2671 | 2677 | 2683 | 2701 | 2707 | 2713 | 2719 | 2725 | 2743 | 2749 | 2755 | 2761 | 2767 | | | | | | | | |
| 2785 | 2791 | 2797 | 2803 | 2809 | 2827 | 2833 | 2839 | 2845 | 2851 | 2875 | 2881 | 2887 | 2893 | 2917 | 2923 | 2929 | 2935 | 2959 | 2965 | 2971 | 2995 | 3001 | | | | | | | | |
| 3007 | 3013 | 3019 | 3037 | 3043 | 3049 | 3055 | 3061 | 3079 | 3085 | 3091 | 3097 | 3121 | 3127 | 3133 | 3139 | 3145 | 3163 | 3169 | 3175 | 3181 | 3187 | 3211 | | | | | | | | |
| 3217 | 3223 | 3229 | 3253 | 3259 | 3265 | 3271 | 3295 | 3301 | 3307 | 3313 | 3331 | 3337 | 3343 | 3349 | 3355 | 3373 | 3379 | 3385 | 3391 | 3397 | 3415 | 3421 | | | | | | | | |
| 3427 | 3433 | 3439 | 3457 | 3463 | 3469 | 3475 | 3481 | 3505 | 3511 | 3517 | 3523 | 3547 | 3553 | 3559 | 3565 | 3589 | 3595 | 3631 | 3637 | 3643 | 3649 | 3667 | | | | | | | | |
| 3673 | 3679 | 3685 | 3691 | 3715 | 3721 | 3727 | 3733 | 3757 | 3763 | 3769 | 3775 | 3799 | 3805 | 3811 | 3817 | 3841 | 3847 | 3853 | 3859 | 3883 | 3889 | 3895 | | | | | | | | |
| 3901 | 3919 | 3925 | 3931 | 3937 | 3943 | 3961 | 3967 | 3973 | 3979 | 3985 | 4003 | 4009 | 4015 | 4021 | 4027 | 4051 | 4057 | 4063 | 4069 | 4093 | 4099 | 4105 | | | | | | | | |
| 4111 | 4135 | 4141 | 4147 | 4153 | 4177 | 4183 | 4189 | 4195 | 4213 | 4219 | 4225 | 4231 | 4237 | 4255 | 4261 | 4267 | 4273 | 4279 | 4303 | 4309 | 4315 | 4321 | | | | | | | | |
| 4345 | 4351 | 4357 | 4363 | 4387 | 4393 | 4399 | 4405 | 4429 | 4435 | 4441 | 4447 | 4471 | 4477 | 4483 | 4489 | 4507 | 4513 | 4519 | 4525 | 4531 | 4555 | 4561 | | | | | | | | |
| 4567 | 4573 | 4591 | 4597 | 4603 | 4609 | 4615 | 4633 | 4639 | 4645 | 4651 | 4657 | 4681 | 4687 | 4699 | 4723 | 4729 | 4735 | 4741 | 4765 | 4771 | 4777 | 4783 | | | | | | | | |
| 4807 | 4813 | 4819 | 4825 | 4849 | 4855 | 4861 | 4867 | 4891 | 4897 | 4903 | 4909 | 4933 | 4939 | 4945 | 4951 | 4969 | 4975 | 4981 | 4987 | 5017 | 5023 | 5029 | | | | | | | | |
| 5035 | 5059 | 5065 | 5071 | 5077 | 5095 | 5101 | 5107 | 5113 | 5119 | 5137 | 5143 | 5149 | 5155 | 5179 | 5185 | 5191 | 5197 | 5203 | 5221 | 5227 | 5233 | 5239 | | | | | | | | |
| 5245 | 5269 | 5275 | 5281 | 5287 | 5311 | 5317 | 5323 | 5329 | 5347 | 5353 | 5359 | 5365 | 5371 | 5395 | 5401 | 5407 | 5413 | 5431 | 5437 | 5443 | 5449 | 5455 | | | | | | | | |
| 5479 | 5485 | 5491 | 5497 | 5515 | 5521 | 5527 | 5533 | 5539 | 5557 | 5563 | 5569 | 5575 | 5581 | 5605 | 5611 | 5617 | 5623 | 5647 | 5653 | 5659 | 5665 | 5689 | | | | | | | | |
| 5695 | 5701 | 5707 | 5725 | 5731 | 5737 | 5743 | 5749 | 5767 | 5773 | 5779 | 5785 | 5791 | 5815 | 5821 | 5827 | 5833 | 5857 | 5863 | 5869 | 5875 | 5899 | 5905 | | | | | | | | |
| 5911 | 5917 | 5935 | 5941 | 5947 | 5953 | 5959 | 5977 | 5983 | 5989 | 5995 | 6001 | 6025 | 6031 | 6037 | 6043 | 6067 | 6073 | 6079 | 6085 | 6109 | 6115 | 6121 | | | | | | | | |
| 6127 | 6151 | 6157 | 6163 | 6169 | 6193 | 6199 | 6205 | 6211 | 6235 | 6247 | 6253 | 6271 | 6277 | 6283 | 6289 | 6295 | 6313 | 6319 | 6325 | 6331 | 6337 | 6355 | | | | | | | | |
| 6361 | 6367 | 6373 | 6379 | 6397 | 6403 | 6409 | 6415 | 6421 | 6445 | 6451 | 6457 | 6463 | 6487 | 6493 | 6499 | 6505 | 6529 | 6535 | 6541 | 6547 | 6565 | 6571 | | | | | | | | |
| 6577 | 6583 | 6589 | 6607 | 6613 | 6619 | 6625 | 6631 | 6655 | 6661 | 6667 | 6673 | 6697 | 6703 | 6709 | 6715 | 6739 | 6745 | 6751 | 6757 | 6781 | 6787 | 6793 | | | | | | | | |
| 6799 | 6817 | 6823 | 6829 | 6835 | 6841 | 6865 | 6871 | 6883 | 6907 | 6913 | 6919 | 6925 | 6949 | 6955 | 6961 | 6967 | 6985 | 6991 | 6997 | 7003 | 7009 | 7033 | | | | | | | | |
| 7039 | 7045 | 7051 | 7075 | 7081 | 7087 | 7093 | 7117 | 7123 | 7129 | 7135 | 7153 | 7159 | 7165 | 7171 | 7177 | 7195 | 7201 | 7207 | 7219 | 7243 | 7249 | 7255 | | | | | | | | |
| 7261 | 7285 | 7291 | 7297 | 7303 | 7327 | 7333 | 7339 | 7345 | 7369 | 7375 | 7381 | 7387 | 7411 | 7417 | 7423 | 7429 | 7447 | 7453 | 7459 | 7465 | 7471 | 7489 | | | | | | | | |
| 7495 | 7501 | 7507 | 7513 | 7531 | 7537 | 7543 | 7549 | 7555 | 7579 | 7585 | 7591 | 7597 | 7621 | 7627 | 7633 | 7639 | 7663 | 7669 | 7675 | 7681 | 7705 | 7711 | | | | | | | | |
| 7717 | 7723 | 7741 | 7747 | 7753 | 7759 | 7765 | 7783 | 7789 | 7795 | 7807 | 7831 | 7837 | 7843 | 7849 | 7873 | 7879 | 7885 | 7891 | 7915 | 7921 | 7927 | 7933 | | | | | | | | |
| 7957 | 7963 | 7975 | 7999 | 8005 | 8011 | 8017 | 8035 | 8041 | 8047 | 8053 | 8059 | 8083 | 8089 | 8095 | 8101 | 8131 | 8137 | 8143 | 8167 | 8173 | 8179 | 8185 | | | | | | | | |
| 8209 | 8215 | 8221 | 8227 | 8251 | 8257 | 8263 | 8269 | 8293 | 8299 | 8305 | 8311 | 8335 | 8341 | 8347 | 8353 | 8371 | 8377 | 8383 | 8389 | 8395 | 8413 | 8419 | | | | | | | | |
| 8425 | 8431 | 8437 | 8461 | 8467 | 8473 | 8479 | 8503 | 8509 | 8515 | 8521 | 8545 | 8551 | 8557 | 8563 | 8587 | 8599 | 8605 | 8629 | 8635 | 8641 | 8647 | 8671 | | | | | | | | |
| 8677 | 8683 | 8689 | 8713 | 8719 | 8725 | 8731 | 8755 | 8761 | 8767 | 8773 | 8797 | 8803 | 8809 | 8815 | 8839 | 8845 | 8851 | 8857 | 8875 | 8881 | 8887 | 8893 | | | | | | | | |
| 8899 | 8917 | 8923 | 8929 | 8935 | 8941 | 8965 | 8971 | 8977 | 8983 | 9007 | 9013 | 9019 | 9025 | 9049 | 9055 | 9061 | 9067 | 9091 | 9097 | 9103 | 9109 | 9133 | | | | | | | | |
| 9139 | 9145 | 9151 | 9175 | 9181 | 9187 | 9193 | 9223 | 9229 | 9235 | 9259 | 9265 | 9271 | 9277 | 9295 | 9301 | 9307 | 9313 | 9319 | 9343 | 9349 | 9355 | 9385 | | | | | | | | |
| 9391 | 9397 | 9403 | 9427 | 9433 | 9439 | 9445 | 9469 | 9475 | 9481 | 9487 | 9511 | 9517 | 9523 | 9553 | 9559 | 9565 | 9571 | 9595 | 9601 | 9607 | 9613 | 9637 | | | | | | | | |
| 9643 | 9649 | 9655 | 9679 | 9685 | 9691 | 9697 | 9721 | 9727 | 9733 | 9739 | 9757 | 9763 | 9769 | 9775 | 9781 | 9799 | 9805 | 9811 | 9817 | 9823 | 9847 | 9853 | | | | | | | | |
| 9859 | 9865 | 9883 | 9889 | 9895 | 9901 | 9907 | 9931 | 9937 | 9943 | 9949 | 9973 | 9979 | 9985 | 9991 | 10015 | 10021 | 10027 | 10033 | 10057 | 10063 | | | | | | | | | | |
| 10069 | 10075 | 10099 | 10105 | 10111 | 10117 | 10135 | 10141 | 10147 | 10153 | 10159 | 10183 | 10189 | 10195 | 10201 | 10225 | 10231 | 10237 | 10243 | | | | | | | | | | | | |
| 10267 | 10273 | 10279 | 10285 | 10309 | 10315 | 10321 | 10327 | 10351 | 10357 | 10363 | 10369 | 10393 | 10399 | 10405 | 10411 | 10435 | 10441 | 10447 | | | | | | | | | | | | |
| 10453 | 10471 | 10477 | 10483 | 10489 | 10495 | 10513 | 10519 | 10525 | 10531 | 10537 | 10561 | 10567 | 10573 | 10579 | 10603 | 10609 | 10615 | 10621 | | | | | | | | | | | | |
| 10645 | 10651 | 10657 | 10687 | 10693 | 10699 | 10705 | 10729 | 10735 | 10741 | 10747 | 10771 | 10783 | 10789 | 10813 | 10819 | 10825 | 10831 | 10855 | | | | | | | | | | | | |
| 10861 | 10867 | 10873 | 10897 | 10903 | 10909 | 10915 | 10939 | 10945 | 10951 | 10957 | 10975 | 10981 | 10987 | 10993 | 10999 | 11017 | 11023 | 11029 | | | | | | | | | | | | |
| 11035 | 11041 | 11065 | 11071 | 11077 | 11083 | 11107 | 11113 | 11119 | 11125 | 11149 | 11155 | 11161 | 11167 | 11191 | 11197 | 11203 | 11209 | 11233 | | | | | | | | | | | | |
| 11239 | 11245 | 11251 | 11275 | 11281 | 11287 | 11293 | 11317 | 11323 | 11329 | 11335 | 11359 | 11365 | 11371 | 11377 | 11407 | 11413 | 11419 | 11443 | | | | | | | | | | | | |
| 11449 | 11455 | 11461 | 11485 | 11491 | 11497 | 11503 | 11527 | 11533 | 11539 | 11545 | 11569 | 11575 | 11581 | 11587 | 11611 | 11617 | 11623 | 11629 | | | | | | | | | | | | |
| 11647 | 11653 | 11659 | 11665 | 11671 | 11695 | 11701 | 11707 | 11713 | 11737 | 11743 | 11749 | 11755 | 11779 | 11785 | 11791 | 11797 | 11815 | 11821 | | | | | | | | | | | | |
| 11827 | 11833 | 11839 | 11863 | 11869 | 11875 | 11881 | 11905 | 11911 | 11917 | 11923 | 11947 | 11953 | 11959 | 11965 | 11989 | 11995 | 12001 | 12007 | | | | | | | | | | | | |
| 12031 | 12037 | 12043 | 12049 | 12073 | 12079 | 12085 | 12091 | 12115 | 12121 | 12127 | 12133 | 12157 | 12163 | 12169 | 12193 | 12199 | 12205 | 12211 | | | | | | | | | | | | |
| 12217 | 12241 | 12247 | 12253 | 12259 | 12277 | 12283 | 12289 | 12295 | 12301 | 12325 | 12331 | 12337 | 12343 | 12367 | 12373 | 12379 | 12385 | 12415 | | | | | | | | | | | | |
| 12421 | 12427 | 12457 | 12463 | 12469 | 12499 | 12505 | 12511 | 12541 | 12547 | 12553 | 12583 | 12589 | 12595 | 12619 | 12625 | 12631 | 12637 | 12661 | | | | | | | | | | | | |
| 12667 | 12673 | 12703 | 12709 | 12715 | 12721 | 12745 | 12751 | 12757 | 12763 | 12787 | 12793 | 12799 | 12805 | 12829 | 12835 | 12841 | 12847 | 12871 | | | | | | | | | | | | |
| 12877 | 12883 | 12889 | 12901 | 12907 | 12931 | 12937 | 12943 | 12973 | 12997 | 13003 | 13009 | 13015 | 13021 | 13057 | 13063 | 13075 | 13081 | 13087 | | | | | | | | | | | | |
| 13093 | 13099 | 13123 | 13129 | 13135 | 13141 | 13165 | 13171 | 13177 | 13207 | 13213 | 13219 | 13225 | 13249 | 13255 | 13261 | 13267 | 13291 | 13297 | | | | | | | | | | | | |
| 13303 | 13309 | 13333 | 13339 | 13345 | 13351 | 13381 | 13387 | 13393 | 13417 | 13423 | 13435 | 13459 | 13465 | 13471 | 13477 | 13501 | 13507 | 13513 | | | | | | | | | | | | |
| 13519 | 13543 | 13549 | 13555 | 13561 | 13585 | 13591 | 13597 | 13603 | 13633 | 13639 | 13645 | 13669 | 13675 | 13681 | 13687 | 13711 | 13717 | 13723 | | | | | | | | | | | | |
| 13753 | 13759 | 13765 | 13771 | 13801 | 13807 | 13813 | 13837 | 13843 | 13849 | 13855 | 13879 | 13885 | 13891 | 13915 | 13927 | 13933 | 13939 | 13969 | | | | | | | | | | | | |
| 13975 | 13981 | 14005 | 14011 | 14017 | 14023 | 14047 | 14053 | 14059 | 14065 | 14089 | 14095 | 14101 | 14107 | 14131 | 14137 | 14143 | 14149 | 14179 | | | | | | | | | | | | |
| 14185 | 14191 | 14221 | 14227 | 14233 | 14257 | 14263 | 14269 | 14275 | 14293 | 14299 | 14305 | 14311 | 14317 | 14341 | 14347 | 14359 | 14383 | 14389 | | | | | | | | | | | | |
| 14395 | 14401 | 14431 | 14437 | 14443 | 14473 | 14479 | 14485 | 14509 | 14515 | 14527 | 14551 | 14557 | 14563 | 14569 | 14593 | 14599 | 14605 | 14611 | | | | | | | | | | | | |
| 14641 | 14647 | 14653 | 14683 | 14689 | 14695 | 14719 | 14725 | 14731 | 14737 | 14761 | 14767 | 14773 | 14779 | 14803 | 14809 | 14815 | 14821 | 14845 | | | | | | | | | | | | |
| 14851 | 14857 | 14863 | 14887 | 14893 | 14899 | 14905 | 14929 | 14935 | 14941 | 14947 | 14971 | 14977 | 14983 | 15013 | 15019 | 15025 | 15031 | 15055 | | | | | | | | | | | | |
| 15061 | 15067 | 15073 | 15097 | 15103 | 15109 | 15115 | 15139 | 15151 | 15157 | 15181 | 15187 | 15193 | 15199 | 15229 | 15235 | 15241 | 15265 | 15271 | | | | | | | | | | | | |
| 15277 | 15283 | 15307 | 15313 | 15319 | 15325 | 15349 | 15355 | 15361 | 15367 | 15391 | 15397 | 15403 | 15409 | 15433 | 15439 | 15445 | 15451 | 15481 | | | | | | | | | | | | |
| 15487 | 15493 | 15517 | 15523 | 15529 | 15535 | 15559 | 15565 | 15571 | 15577 | 15607 | 15619 | 15643 | 15649 | 15655 | 15661 | 15691 | 15697 | 15733 | | | | | | | | | | | | |
| 15739 | 15745 | 15775 | 15781 | 15787 | 15811 | 15817 | 15823 | 15829 | 15853 | 15859 | 15865 | 15871 | 15901 | 15907 | 15913 | 15949 | 15955 | 15985 | | | | | | | | | | | | |
| 15991 | 15997 | 16021 | 16027 | 16033 | 16039 | 16063 | 16075 | 16105 | 16111 | 16117 | 16123 | 16147 | 16159 | 16165 | 16189 | 16201 | 16207 | 16231 | | | | | | | | | | | | |
| 16243 | 16249 | 16279 | 16285 | 16291 | 16327 | 16333 | 16357 | 16369 | 16375 | 16399 | 16411 | 16417 | 16441 | 16453 | 16459 | 16483 | 16489 | 16495 | | | | | | | | | | | | |
| 16501 | 16525 | 16531 | 16537 | 16543 | 16573 | 16579 | 16585 | 16609 | 16615 | 16621 | 16627 | 16651 | 16657 | 16663 | 16669 | 16699 | 16705 | 16711 | | | | | | | | | | | | |
| 16747 | 16753 | 16777 | 16789 | 16795 | 16819 | 16825 | 16831 | 16837 | 16867 | 16873 | 16879 | 16903 | 16909 | 16915 | 16921 | 16945 | 16951 | 16957 | | | | | | | | | | | | |
| 16963 | 16987 | 16993 | 16999 | 17029 | 17041 | 17047 | 17071 | 17077 | 17083 | 17089 | 17113 | 17119 | 17125 | 17131 | 17155 | 17161 | 17167 | 17197 | | | | | | | | | | | | |
| 17209 | 17215 | 17239 | 17245 | 17251 | 17257 | 17281 | 17287 | 17293 | 17299 | 17323 | 17329 | 17335 | 17341 | 17371 | 17377 | 17383 | 17407 | 17413 | | | | | | | | | | | | |
| 17419 | 17425 | 17455 | 17461 | 17467 | 17497 | 17503 | 17509 | 17533 | 17539 | 17545 | 17581 | 17587 | 17593 | 17617 | 17623 | 17629 | 17635 | 17659 | | | | | | | | | | | | |
| 17665 | 17671 | 17677 | 17707 | 17713 | 17749 | 17755 | 17761 | 17785 | 17791 | 17803 | 17827 | 17833 | 17839 | 17845 | 17869 | 17881 | 17887 | 17911 | | | | | | | | | | | | |
| 17923 | 17929 | 17965 | 17971 | 17995 | 18001 | 18007 | 18013 | 18049 | 18055 | 18079 | 18085 | 18091 | 18121 | 18127 | 18133 | 18139 | 18163 | 18175 | | | | | | | | | | | | |
| 18181 | 18205 | 18217 | 18223 | 18247 | 18253 | 18259 | 18265 | 18289 | 18295 | 18301 | 18307 | 18337 | 18343 | 18349 | 18373 | 18379 | 18385 | 18391 | | | | | | | | | | | | |
| 18415 | 18421 | 18427 | 18433 | 18457 | 18463 | 18469 | 18475 | 18499 | 18511 | 18517 | 18541 | 18547 | 18553 | 18559 | 18583 | 18589 | 18595 | 18601 | | | | | | | | | | | | |
| 18625 | 18631 | 18637 | 18643 | 18667 | 18673 | 18679 | 18685 | 18709 | 18715 | 18721 | 18727 | 18751 | 18757 | 18763 | 18769 | 18793 | 18805 | 18811 | | | | | | | | | | | | |
| 18847 | 18853 | 18883 | 18925 | 18931 | 18973 | 19009 | 19015 | 19051 | 19057 | 19093 | 19177 | 19183 | 19219 | 19261 | 19591 | 19597 | 19603 | 19609 | | | | | | | | | | | | |
| 19633 | 19639 | 19645 | 19651 | 19687 | 19693 | 19717 | 19729 | 19735 | 19759 | 19765 | 19771 | 19777 | 19801 | 19807 | 19819 | 19843 | 19849 | 19855 | | | | | | | | | | | | |
| 19861 | 19885 | 19891 | 19897 | 19903 | 19927 | 19939 | 19945 | 19969 | 19981 | 19987 | 20011 | 20017 | 20023 | 20029 | 20053 | 20059 | 20065 | 20071 | | | | | | | | | | | | |
| 20101 | 20107 | 20113 | 20149 | 20155 | 20179 | 20185 | 20191 | 20197 | 20221 | 20227 | 20233 | 20239 | 20263 | 20269 | 20275 | 20305 | 20311 | 20317 | | | | | | | | | | | | |
| 20323 | 20347 | 20353 | 20359 | 20365 | 20395 | 20401 | 20407 | 20431 | 20437 | 20443 | 20473 | 20479 | 20485 | 20491 | 20521 | 20527 | 20533 | 20557 | | | | | | | | | | | | |
| 20569 | 20575 | 20599 | 20605 | 20611 | 20617 | 20641 | 20647 | 20653 | 20659 | 20683 | 20689 | 20695 | 20701 | 20725 | 20731 | 20737 | 20743 | 20767 | | | | | | | | | | | | |
| 20773 | 20779 | 20785 | 20809 | 20815 | 20821 | 20827 | 20851 | 20863 | 20869 | 20905 | 20911 | 20941 | 20947 | 20989 | 21025 | 21031 | 21067 | 21109 | | | | | | | | | | | | |
| 21115 | 21151 | 21157 | 21235 | 21277 | 21319 | 21361 | 21733 | 21745 | 21751 | 21781 | 21787 | 21817 | 21823 | 21829 | 21835 | 21859 | 21865 | 21871 | | | | | | | | | | | | |
| 21877 | 21901 | 21907 | 21913 | 21943 | 21949 | 21955 | 21961 | 21985 | 21991 | 22003 | 22027 | 22033 | 22039 | 22045 | 22069 | 22075 | 22081 | 22087 | | | | | | | | | | | | |
| 22111 | 22117 | 22123 | 22129 | 22159 | 22165 | 22171 | 22201 | 22207 | 22213 | 22237 | 22249 | 22273 | 22285 | 22291 | 22297 | 22321 | 22327 | 22333 | | | | | | | | | | | | |
| 22339 | 22363 | 22369 | 22375 | 22381 | 22405 | 22411 | 22417 | 22423 | 22453 | 22459 | 22489 | 22495 | 22501 | 22507 | 22531 | 22537 | 22543 | 22549 | | | | | | | | | | | | |
| 22573 | 22579 | 22585 | 22591 | 22627 | 22657 | 22669 | 22675 | 22699 | 22705 | 22711 | 22717 | 22747 | 22753 | 22783 | 22795 | 22801 | 22825 | 22831 | | | | | | | | | | | | |
| 22837 | 22843 | 22867 | 22873 | 22879 | 22885 | 22909 | 22921 | 22927 | 22951 | 22957 | 22963 | 22969 | 22999 | 23005 | 23011 | 23041 | 23047 | 23053 | | | | | | | | | | | | |
| 23077 | 23095 | 23119 | 23125 | 23131 | 23137 | 23161 | 23167 | 23173 | 23203 | 23215 | 23221 | 23251 | 23293 | 23299 | 23305 | 23335 | 23341 | 23347 | | | | | | | | | | | | |
| 23371 | 23377 | 23383 | 23389 | 23419 | 23425 | 23431 | 23467 | 23473 | 23497 | 23509 | 23515 | 23539 | 23545 | 23551 | 23587 | 23593 | 23623 | 23629 | | | | | | | | | | | | |
| 23635 | 23641 | 23665 | 23671 | 23677 | 23683 | 23707 | 23719 | 23749 | 23761 | 23767 | 23791 | 23803 | 23839 | 23845 | 23881 | 23887 | 23893 | 23917 | | | | | | | | | | | | |
| 23929 | 23935 | 23959 | 23965 | 23971 | 24001 | 24007 | 24013 | 24043 | 24055 | 24061 | 24085 | 24091 | 24097 | 24127 | 24133 | 24139 | 24169 | 24175 | | | | | | | | | | | | |
| 24187 | 24217 | 24223 | 24229 | 24259 | 24265 | 24271 | 24307 | 24313 | 24355 | 24379 | 24385 | 24391 | 24433 | 24469 | 24475 | 24481 | 24511 | 24517 | | | | | | | | | | | | |
| 24553 | 24589 | 24595 | 24601 | 24631 | 24637 | 24673 | 24685 | 24721 | 24805 | 24841 | 24847 | 24883 | 24925 | 24967 | 25057 | 25063 | 25099 | 25105 | | | | | | | | | | | | |
| 25141 | 25147 | 25153 | 25177 | 25189 | 25219 | 25231 | 25261 | 25273 | 25303 | 25309 | 25315 | 25345 | 25351 | 25393 | 25399 | 25429 | 25435 | 25441 | | | | | | | | | | | | |
| 25447 | 25471 | 25483 | 25489 | 25513 | 25525 | 25531 | 25561 | 25567 | 25573 | 25597 | 25609 | 25645 | 25651 | 25657 | 25687 | 25693 | 25723 | 25729 | | | | | | | | | | | | |
| 25735 | 25765 | 25771 | 25777 | 25807 | 25813 | 25819 | 25849 | 25861 | 25891 | 25897 | 25903 | 25933 | 25939 | 25945 | 25981 | 25987 | 26017 | 26029 | | | | | | | | | | | | |
| 26059 | 26065 | 26071 | 26101 | 26107 | 26113 | 26143 | 26149 | 26155 | 26185 | 26191 | 26197 | 26233 | 26239 | 26275 | 26281 | 26317 | 26323 | 26359 | | | | | | | | | | | | |
| 26371 | 26401 | 26407 | 26437 | 26449 | 26479 | 26485 | 26491 | 26497 | 26527 | 26569 | 26575 | 26611 | 26617 | 26623 | 26647 | 26653 | 26659 | 26695 | | | | | | | | | | | | |
| 26701 | 26707 | 26731 | 26743 | 26749 | 26785 | 26827 | 26869 | 26875 | 26905 | 26911 | 26941 | 26947 | 26953 | 26983 | 26995 | 27001 | 27025 | 27037 | | | | | | | | | | | | |

Table 14: Table of 3960 possible exceptions in the PBD-closure of $\{7, 13\}$ (cont.)

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 27043 | 27067 | 27079 | 27109 | 27115 | 27121 | 27127 | 27151 | 27163 | 27169 | 27193 | 27199 | 27205 | 27211 | 27247 | 27253 | 27277 | 27289 | 27319 |
| 27331 | 27337 | 27373 | 27409 | 27415 | 27421 | 27451 | 27463 | 27493 | 27499 | 27529 | 27535 | 27541 | 27547 | 27577 | 27613 | 27619 | 27625 | 27631 |
| 27661 | 27667 | 27697 | 27709 | 27745 | 27751 | 27793 | 27835 | 27865 | 27877 | 27919 | 27949 | 27961 | 27997 | 28039 | 28045 | 28087 | 28117 | 28123 |
| 28129 | 28159 | 28165 | 28171 | 28201 | 28213 | 28243 | 28255 | 28291 | 28333 | 28375 | 28417 | 28459 | 28465 | 28471 | 28507 | 28513 | 28537 | 28579 |
| 28585 | 28627 | 28669 | 28711 | 28747 | 28789 | 28999 | 29005 | 29167 | 29209 | 29215 | 29251 | 29293 | 29713 | 29719 | 29725 | 29755 | 29761 | 29845 |
| 29851 | 29893 | 29923 | 29935 | 29941 | 29965 | 29977 | 29983 | 30007 | 30019 | 30055 | 30097 | 30139 | 30145 | 30175 | 30181 | 30193 | 30217 | 30223 |
| 30229 | 30235 | 30259 | 30271 | 30277 | 30301 | 30397 | 30427 | 30469 | 30475 | 30481 | 30511 | 30553 | 30565 | 30637 | 30643 | 30685 | 30721 | 30727 |
| 30763 | 30805 | 30847 | 30895 | 30931 | 30937 | 30979 | 31021 | 31099 | 31105 | 31111 | 31141 | 31183 | 31189 | 31225 | 31231 | 31273 | 31309 | 31315 |
| 31351 | 31393 | 31399 | 31477 | 31519 | 31537 | 31561 | 31567 | 31573 | 31609 | 31615 | 31645 | 31651 | 31657 | 31663 | 31687 | 31693 | 31705 | 31729 |
| 31741 | 31771 | 31777 | 31819 | 31861 | 31897 | 31909 | 31939 | 31945 | 31951 | 31957 | 31987 | 31993 | 32023 | 32029 | 32035 | 32041 | 32065 | 32107 |
| 32113 | 32155 | 32161 | 32167 | 32191 | 32197 | 32203 | 32233 | 32239 | 32245 | 32251 | 32287 | 32293 | 32317 | 32329 | 32335 | 32359 | 32365 | 32371 |
| 32401 | 32407 | 32449 | 32455 | 32461 | 32485 | 32491 | 32497 | 32527 | 32533 | 32539 | 32569 | 32575 | 32581 | 32587 | 32611 | 32653 | 32659 | 32665 |
| 32695 | 32701 | 32749 | 32755 | 32779 | 32785 | 32791 | 32821 | 32827 | 32833 | 32863 | 32869 | 32875 | 32881 | 32905 | 32917 | 32923 | 32947 | 32953 |
| 32959 | 32989 | 32995 | 33031 | 33037 | 33079 | 33085 | 33091 | 33121 | 33127 | 33133 | 33157 | 33163 | 33169 | 33175 | 33199 | 33211 | 33253 | 33289 |
| 33331 | 33337 | 33373 | 33379 | 33415 | 33421 | 33457 | 33499 | 33505 | 33547 | 33589 | 33625 | 33631 | 33667 | 33673 | 33715 | 33757 | 33919 | 33961 |
| 34171 | 34213 | 34585 | 34591 | 34597 | 34603 | 34627 | 34633 | 34639 | 34669 | 34675 | 34681 | 34687 | 34711 | 34717 | 34723 | 34753 | 34759 | 34807 |
| 34813 | 34837 | 34849 | 34879 | 34885 | 34891 | 34927 | 34933 | 34963 | 35005 | 35011 | 35017 | 35047 | 35053 | 35059 | 35089 | 35095 | 35101 | 35131 |
| 35137 | 35179 | 35185 | 35191 | 35221 | 35227 | 35233 | 35257 | 35275 | 35299 | 35305 | 35311 | 35317 | 35341 | 35347 | 35383 | 35425 | 35431 | 35443 |
| 35467 | 35473 | 35479 | 35485 | 35509 | 35515 | 35521 | 35527 | 35551 | 35557 | 35593 | 35599 | 35677 | 35689 | 35695 | 35719 | 35725 | 35767 | 35773 |
| 35803 | 35809 | 35815 | 35821 | 35845 | 35851 | 35857 | 35863 | 35887 | 35893 | 35899 | 35929 | 35935 | 35941 | 35947 | 35971 | 35977 | 35983 | 35989 |
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